THE SOCIAL VALUE OF INFORMATION UNCERTAINTY

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ABSTRACT. We consider information acquisition uncertainty in the Grossman-Stiglitz economy and show that costly information in financial markets can be welfare improving. The marginal welfare can be decomposed into a positive information-gain effect and a negative informed-trading effect. The welfare benefit can be substantial for speculators who provide liquidity when risk-sharing incentives are weak and information quality is moderate so that the information-gain effect dominates. When the informed-trading effect dominates, only no-information equilibrium is Pareto-optimal. With heterogenous endowment shocks, the Hirshleifer effect allows for a continuum of Pareto optimal equilibria. This suggests that regulatory efforts to alter the amount of informed trading may be unnecessary.

Key words: Social welfare, rational expectations equilibrium, informed trading, information acquisition uncertainty, probabilistic choices.

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1. Introduction

Is costly information acquisition beneficial or harmful to investors' welfare? Since the introduction of rational expectations equilibrium (REE) models (Grossman and Stiglitz, 1980; Hellwig, 1980; Diamond and Verrecchia, 1981; Admati, 1985), there has been an extensive debate on whether spending resources on acquiring private information about the future payoff of a risky asset has any social value. In other words, when focusing on the overall welfare, should the regulators aim to create a level playing field by restricting traders from gathering private information?

We try to address this question in a canonical REE model, with one key difference in how investors make information acquisition decisions. Instead of deciding whether or not to pay a fixed cost for purchasing a private signal about asset's fundamentals, investors in our model make *probabilistic choices*, i.e., they pay a variable cost in exchange for an optimal probability of being informed. The key point we want to raise is that information about fundamentals needs to be discovered. Investors must spend effort (hence, money) to find relevant information on financial markets, and even in this case, they do not necessarily succeed eventually. The simplest way to model this paradigm is to assume that each investor optimally allocates part of her wealth to increase the probability of being informed.

Information acquisition under probabilistic choices is similar to playing a lottery game: the more tickets a player purchases, the better the chance of winning, and the higher is the cost of the action. In the context of financial markets, an investor may purchase, for example, an analyst report, hoping to obtain valuable information about the fundamental value of a firm. *Ex-ante*, investors expect a higher gain by paying more for the report. *Ex-post*, the analyst report could turn out to be either informative or completely useless. Think about a continuum of information providers of different reliability: the more the investor pays for a provider with better quality, the higher the chance to be informed. Put differently, each investor strategically chooses the information provider to maximize expected utility. In the standard Grossman-Stiglitz economy, where traders chose to be either informed (paying a fixed cost) or to remain uninformed, it is well-known that information acquisition always reduces welfare. In contrast, we find that such costly probabilistic information acquisition can be welfare-improving.

To provide intuition about this finding, we consider the following illustrative example to compare settings with and without information acquisition uncertainty.

Example. A trader faces two outcomes; (i) be informed and receive a utility (or payoff) x, or (ii) be uninformed and receive a utility (payoff) $y < x$, in two scenarios;

- Scenario 1 (Binary Choice) – the trader can pay a fixed cost c to be informed with certainty.
- Scenario 2 (Probabilistic Choice) – the trader can pay a quadratic cost $\mu\, p^2$ ($\mu > 0$) for a probability p to be informed so as to maximize $\mathcal{U}(p) = p\, x + (1-p)\, y - \mu\, p^2$.

In Scenario 1, three outcomes are possible. Either $x-c > y$ and the trader chooses to pay c; $x - c < y$ and the trader chooses not to pay c; or $x - c = y$ in which case the trader is indifferent. Typically, models with endogenous information acquisition consider the third case as the overall equilibrium, where the market fraction of informed traders, λ, is endogenously determined by $c = x(\lambda) - y(\lambda)$ in equilibrium. Lowering the cost allows more investors to be informed, but reduces the relative benefit (and hence incentive) to become informed, meaning $x'(\lambda) - y'(\lambda) < 0$, i.e., the market exhibits strategic substitutability in information acquisition. In terms of welfare implications, since traders' welfare in equilibrium is given by $\mathcal{W}(\lambda) = y(\lambda)$, whether information acquisition is beneficial or harmful depends on the sign of $y'(\lambda)$. There are at least two reasons to believe that $y'(\lambda) < 0$ (and hence $x'(\lambda) < 0$), so that more informed trading reduces the utility (payoffs), both of which are related to the *destruction of trading opportunities* due to information disclosure by informed trading.

In the standard Grossman-Stiglitz economy, the key idea is that informed trading resolves payoff uncertainty, distorts risk-sharing amongst traders and thus reduces welfare through two channels. The first channel is the well-known *Hirshleifer effect* (Hirshleifer, 1971). Suppose traders want to hedge against future endowment shocks by trading the risky asset. By resolving future payoff uncertainty, informed trading brings the asset price closer to its fundamental, which makes it more difficult to insure against the realization of the asset payoff, and thus reduces traders' willingness to trade. The second channel is the *return effect* (Kurlat & Veldkamp, 2015). In this case, even without endowment shocks, more informed trading is still harmful to traders, because the reduction in risk also reduces an asset's expected return, and the net effect on welfare is negative. As explained by Goldstein and Yang (2017), a "*common theme of both channels is that disclosure harms investors through destroying trading opportunities*". Therefore, both the Hirshleifer and return effects contribute negatively to marginal welfare. This is now well understood, and is commonly referred to as the *informed-trading effect*.

We now turn to Scenario 2, in which traders make probabilistic information choices. The optimal probability a trader chooses is $p^* = (x-y)/(2\mu)$, with trader's welfare $\mathcal{W} \equiv \mathcal{U}(p^*) = y + p^*(x-y)/2 > y$.[1] Therefore, under probabilistic choices a trader is always better off at the optimum compared to staying uninformed, which contributes positively to marginal welfare. We refer to this novel effect as the anticipatory *information-gain effect*. More explicitly, the payoffs x and y depend on the optimal probability p^* and hence on the level of informed trading λ. In equilibrium, $p^*(\lambda) = \lambda = [x(\lambda) - y(\lambda)]/(2\mu)$ and $\mathcal{W}(\lambda) \equiv \mathcal{U}(p^*, \lambda) = y(\lambda) + p^*[x(\lambda) - y(\lambda)]/2$. In this case, the marginal welfare can be decomposed into

$$\mathcal{W}'(\lambda) = \frac{\partial \mathcal{U}(p^*, \lambda)}{\partial p^*} + \frac{\partial \mathcal{U}(p^*, \lambda)}{\partial \lambda} = \underbrace{\frac{1}{2}[x(\lambda) - y(\lambda)]}_{\text{information-gain effect}} + \underbrace{y'(\lambda) + \frac{1}{2}\lambda(x'(\lambda) - y'(\lambda))}_{\text{informed-trading effect}}.$$

Therefore, costly information acquisition can be welfare-improving when the positive information-gain effect dominates the negative informed-trading effect. Intuitively, the information-gain effect comes from traders, who make optimal probabilistic information choices, anticipating the potential benefit that can be realized if they succeed in acquiring the private signal and thus become informed. This anticipatory welfare effect is non-existent in the Grossman-Stiglitz model since traders already know the outcome of their information choice once they decide to pay or not pay the cost.

In the spirit of Grossman and Stiglitz (1980), our aim is to use a REE model to analyze the trade-off between the information-gain and informed-trading effects

[1]Note that in case $(x-y)/(2\mu) \geq 1$, then p^* is settled to 1.

on marginal welfare and examine the market conditions under which information acquisition is welfare-improving. Section 2 presents a noise-trader model in line with Grossman and Stiglitz (1980), where a continuum of identical CARA utility-maximizing traders (speculators) provide liquidity to noise traders whose supplies are exogenous and normally distributed. The model can be separated into two stages. In the first stage, each trader strategically chooses a probability p to be informed and pays the corresponding cost $\mu\, c(p)$. Facing information acquisition uncertainty, the investor's objective is to choose the optimal probability of observing the private signal. We study how these probabilistic information choices change the equilibrium outcome and lead to different welfare implications. As a result, a fraction, λ, of the traders will be informed by receiving a private signal about the asset's future payoff. In the second stage, each trader forms an optimal portfolio conditional on his information set and the equilibrium price is determined by the market clearing condition in financial market. We fully characterize a unique equilibrium in terms of the price for the risky asset and fraction of informed traders.

In Section 3, we analyze the effect of traders' probabilistic information choices on their collective welfare, and obtain three main results. First, we explicitly characterize the informed-trading and information-gain effects and derive a necessary and sufficient condition for a positive marginal welfare under information acquisition, i.e., *a marginal increase in the fraction of informed traders leads to an increase in their welfare*. We find that welfare-improvement is more likely to occur when risk-sharing incentives and information quality are both low. Notably, given that price information efficiency improves with information acquisition, this also implies that information acquisition uncertainty can offer a solution to the paradox in the Grossman-Stiglitz economy that information acquisition improves price efficiency but reduces welfare. Intuitively, higher information quality (e.g., a more precise private signal) means that more uncertainty is resolved by informed trading; while higher risk-sharing incentives (e.g., a higher risk aversion) means traders care more about their trading opportunities. Both of them worsen the negative informed-trading effect on marginal welfare. Secondly, there exists a unique *Pareto-optimal* state (λ^*) where traders' welfare is maximized. This implies that an increase in the level of informed trading improves welfare for $\lambda < \lambda^*$. Therefore, from a policymaker's point of view, whether or not a policy attempting to change the level of informed trading by tightening or loosening restrictions is welfare-improving depends on the current market state. More specifically, a policymaker can control the level of informed trading by fine-tuning the cost sensitivity parameter, i.e., μ as defined in the example. Thirdly, the magnitude of the welfare benefit of information acquisition is highly significant, especially when the risk-sharing incentives are low. As a result, completely removing the opportunity to acquire private information can potentially incur a large welfare cost.

As pointed out by Bond and Garcia (2018), welfare analysis can be compromised when the demand of the noise traders is not explicitly modeled. To overcome this issue, we present a second formulation of the model in Section 4, where the noise demand is endogenized. In this setup, traders' optimal demand consists of a speculative component and a hedging component related to the presence of a trader-specific endowment shock. In this more general setting, in addition to the information-gain effect, we are able to capture both the aforementioned Hirshleifer and return effects, which together characterize the negative informed-trading effect on marginal welfare. We find that information acquisition affects traders' welfare differently depending on the size of their endowment shocks. It tends to be welfare-improving for liquidity providers, i.e., traders with small endowment shocks who trade to make speculative profits, but welfare-reducing for liquidity consumers, i.e., traders with large endowment shocks who trade to hedge their endowment risk. This is because the Hirshleifer effect dominates marginal welfare for those traders who are liquidity consumers with large endowment shocks. The policy implications from analyzing information acquisition in the more general endowment economy are twofold: either the no-information equilibrium is the only Pareto-optimal equilibrium or there exists a continuum of Pareto optimal equilibria. This latter situation happens when risk-sharing incentives are low and the informed-trading effect does not dominate the information-gain effect. In comparison, when risk-sharing incentives are high and the informed-trading effect dominates, only the no-information equilibrium, i.e., $\lambda = 0$, is Pareto-optimal.

We conclude the paper in Section 5 and collect all the proofs and additional discussions in Appendices A and B.

Related Literature. Our paper is closely related to the vast and diverse literature that examines the equilibrium outcomes of endogenous information acquisition and disclosure and their implications to welfare, going back at least to Hirshleifer (1971).

Our first contribution to the literature offers a novel channel through which information acquisition improves investors' *ex-ante* welfare in financial markets. In the Grossman-Stiglitz setup, it is well-understood that, by resolving uncertainty and destroying trading opportunities, information disclosure in financial markets is welfare reducing (see, e.g., Allen (1984), Kurlat and Veldkamp (2015), the excellent survey paper of Goldstein and Yang (2017) and the references cited therein). For welfare improvement, some authors have proposed possible market regulations, including to impose a tax on information gathering (Allen, 1984) or a mandatory information disclosure (Kurlat and Veldkamp, 2015).[2] Other literature has identified alternative channels.[3] The present paper deviates from the literature by introducing information acquisition uncertainty to the Grossman-Stiglitz equilibrium. This is important for market regulators who are concerned about transparency and having a level playing field with respect to information in financial markets. According to Glosten and Putnins (2019), "*Regulation can alter the amount of informed trade. For example, regulation can affect the costs of private information, make it more or less accessible, or prohibit the use of particular types of information*". Our analysis suggests the following. If we only consider the welfare of the liquidity providers (speculators) in the noise-trader model, a low level of informed trading can potentially be Pareto-optimal, especially when risk-sharing incentives is low and information quality is moderate. In the more general setup with endowment shocks, such costly financial reforms may not be necessary, since any changes to cost sensitivity may

[2]By examining the welfare implication of mandatory disclosure by asset issuers to potential buyers about asset quality, Kurlat and Veldkamp (2015) find that, even when asset issuers bear of the cost of information and providing information improves risk allocation, information acquisition can still be welfare-reducing, "*simply because resolving risk reduces returns*".

[3]They include risk-sharing among outsiders with stochastic liquidity (Bhattacharya and Nicodano, 2001), the feedback effect of investment policy (Dow and Rahi, 2003), preventing market failure (Goldstein and Leitner, 2018), externality in the use of private information (Vives, 2017), and heterogeneous private valuations of risky assets (Rahi and Zigrand, 2018; Rahi, 2021).

be beneficial for liquidity providers (speculators) and welfare harmful for liquidity consumers (hedgers), and therefore not Pareto-improving. An exception is when the current level of informed trading or the risk-sharing incentives are too high, in which case increasing cost to discourage information gathering can improve welfare for all traders.

Secondly, this paper complements to the literature that examines the impact of public information transparency on welfare. This literature has provided possible reasons for the negative welfare effect of public information. By introducing a beauty-contest motive into agents' preferences in a coordination game, Morris and Shin (2002) show that a noisy public signal can be detrimental to *ex-ante* welfare. Angeletos and Pavan (2007) find that whether increased reliance on public information is socially valuable depends not only on the form of strategic interaction, but also on the type of economy and information structure. Amador and Weill (2010) study the effect of public announcements on price informativeness and welfare (in a Lucas (1972) model with random productivity and nominal shocks). They find that public information release may increase uncertainty and reduce welfare and the optimal communication policy is always to release either all or none of the information. To prevent the negative welfare effects of public information, Morris and Shin (2002) propose to withhold relevant information or deliberately reduce information precision. By distinguishing between precision of information and degree of publicity, Cornand and Heinemann (2008) show that information released with maximum precision can be welfare-enhancing if only provided to some fraction of market participants. Contrary to allowing for intermediate degrees of publicity exogenously in Cornand and Heinemann (2008), we endogenize the level of information release as an agents' decision variable through a strategic game: agents maximize the likelihood of being informed *ex-post* by paying a cost which also depends on this decision variable. It turns out that, at the endogenous information equilibrium, there is a trade-off between information quality and degree of publicity for Pareto optimality resembling the findings in Morris and Shin (2002) and Cornand and Heinemann (2008). We find that, to maximize traders' welfare, policymakers can increase information acquisition cost sensitivity in response to high information quality in order to drive down informed trading to the Pareto-optimal state, however, not necessarily to the no-information equilibrium. Moreover, with an intermediate level of information quality and low risk-sharing incentives, the relative welfare improvement from the no-information equilibrium can be very significant. In other words, there can be a large welfare loss for the liquidity providers in the market if the information acquisition opportunity was completely lost.

Thirdly, this paper relates to a separate strand of literature which studies the complementarity of information acquisition and the effect of information frictions on price efficiency. For example, Veldkamp (2006) shows that competitive information production can lead to information complementarity and inflate asset prices and volatility. Goldstein and Yang (2015) extend the Grossman-Stiglitz model to include two dimensions of uncertainty to analyze the interaction between different types of informed traders. Breugem and Buss (2019) show that benchmarking behavior of the institutional investors can lead to a reduction in information acquisition and price efficiency. We show that information acquisition uncertainty can simultaneously improve *ex-ante* welfare and price efficiency when the level of informed trading and risk-share incentives are sufficiently low. In comparison, when risk-sharing incentives are too high only the no-information equilibrium, in which price reveals no private information, is Pareto-optimal.

Fourthly, this paper develops a new modelling framework for information acquisition in financial markets. Our probabilistic choice model is adopted from Mattsson and Weibull (2002). In their model, an individual optimally makes an effort to deviate from the status-quo and changes the likelihood of a finite set of possible scenarios in order to get closer to implementing a more desired outcome.[4] In our context, the desired outcome is to observe the private signal and be informed. In contrast to Mattsson and Weibull (2002), we use monetary instead of utility cost, which is more suited to the Grossman-Stiglitz framework. Moreover, we use a quadratic instead of an entropic cost function, which allows us to obtain more explicit results without sacrificing any economic insights. The framework naturally connects

[4]The framework developed in Mattsson and Weibull (2002) can also be related to rational inattention as described in Sims (2003), where economic agents have limited ability to process or pay attention to information.

settings with and without information acquisition uncertainty, providing economic channels explicitly for welfare improvement.

Finally, the costly information game setup in this paper also contributes to the large literature on endogenous information equilibrium with rational expectations. Starting from Diamond and Verrecchia (1981), Admati (1985), Diamond (1985), and Admati and Pfleiderer (1987), there have been intensive studies on information markets. When an increase in information demand causes more information to be provided at a lower price, Veldkamp (2006) identifies information markets where the equilibrium level of information emerges endogenously as the source of media frenzy and market herding.[5] In contrast, we consider an increasing and convex cost function for information that depends on the probability of being informed. In some sense, the randomness on what investors place their bets is not the quality of the signal, rather the quality of the information provider. The resulting trade-off between the equilibrium level of informed trading and information quality for Pareto optimality is in line with rational inattention described in Sims (2003): economic agents have limited ability to process information or to pay attention to it. Although the rational inattention literature is mainly focused on information precision, our agents are also characterized by rational inattention. In particular, with increasing information complexity, they are aware of the limited resources to grasp information and set the optimal level of effort (reduction of inattention). The greater the agent's effort, the higher the probability to be informed, and the higher is the attention the agent puts on available signals.

Interestingly, Hoff and Stiglitz (2016) recently discussed the importance of advancing the economic modelling background to allow for endogenization of preferences and behaviors. They argue that an equilibrium in the economy is a joint (endogenous) outcome expressed in terms of *probability of types* and *market prices*. In this respect, our framework can be seen as an attempt to introduce *endogenization of types* into an otherwise standard exchange economy.

[5]Hellwig and Veldkamp (2009) further discuss information acquisition in a Morris-Shin "beauty contest" framework, where signals can be bought by a heterogeneous population of strategic agents for a cost depending on the quality of the signal.

2. A Noise-Trader Model

This section extends the standard Grossman-Stiglitz model to incorporate information acquisition uncertainty under probabilistic choices. There is a continuum of identical (price-taking) constant absolute risk aversion (CARA) utility-maximizing traders, indexed by $i \in (0,1)$, who can invest in a risk-free asset (with a normalized interest rate of zero) and a risky asset. The risky asset has a payoff $\tilde{D} = \tilde{\theta} + \tilde{\epsilon}$, where $\tilde{\theta} \sim \mathcal{N}(0, v_\theta)$ is a private signal about the payoff, and $\tilde{\epsilon} \sim N(0, v_\epsilon)$ is the residual noise, which is independent of $\tilde{\theta}$. Thus, $\tilde{D} \sim \mathcal{N}(0, v_D)$, where $v_D = v_\theta + v_\epsilon$, and $\tilde{D}|\theta \sim \mathcal{N}(\theta, v_\epsilon)$.

2.1. **Probabilistic Choice and Trading.**

There are three dates, $t = 0, 1, 2$. At $t = 0$, trader i chooses a probability p_i^* to observe the private signal $\tilde{\theta}$ at a cost $\mu\, c(p_i^*)$, where $c(p)$ is an increasing and convex cost function with $c(0) = 0$ and $\mu > 0$ measures the *cost sensitivity*. At $t = 1$, a Boolean random variable $\tilde{\omega}_i$ is drawn independently for each trader with $\mathbb{P}(\tilde{\omega}_i = 1) = p_i^*$ and $\mathbb{P}(\tilde{\omega}_i = 0) = 1 - p_i^*$. If $\tilde{\omega}_i = 1$, the trader observes $\tilde{\theta}$ and becomes *informed* (type I). Otherwise, $\tilde{\omega}_i = 0$; the trader does not observe $\tilde{\theta}$ and remains *uninformed* (type U).

Let $\tilde{P}$ be the price of the risky asset. Depending on his type, each trader chooses his optimal demand x_i^* in the risky asset. Assume all utility-maximizing traders have a CARA coefficient α and zero initial wealth. Then trader i's objective is to choose p_i and x_i (conditional on his information $\mathcal{F}_i$) to maximize

$$\mathbb{E}\Big[- e^{-\alpha\left(x_i \tilde{R} - \mu\, c(p_i)\right)} \Big| \mathcal{F}_i \Big], \qquad \tilde{R} \equiv \tilde{D} - \tilde{P}. \tag{2.1}$$

In addition, there is also a group of noise traders whose net supply for the risky asset is $\tilde{z} \sim N(0, v_z)$, which is independent of $\tilde{\theta}$ and $\tilde{\epsilon}$.[6] At $t = 2$, the noise supply $\tilde{z}$ is realized, $\tilde{P}$ is determined by the market clearing condition, and each trader receives his allocation of shares in the risky asset. Then the payoff $\tilde{D}$ is realized and consumption occurs.

[6] In Section 4, we model the behaviour of liquidity (or noise) traders explicitly using endowment shocks. To simplify the analysis, we assume a noisy supply in the baseline model in Sections 2 and 3.

2.2. **Portfolio Choice and Financial Market Equilibrium.** The CARA-normal setting implies that traders' portfolio choices at $t = 1$ follow the standard mean-variance criteria. Hence, if we take the *fraction of informed traders* λ as a given *state variable*, representing the *level of informed trading*, we can solve for equilibrium in the financial market in each market state.

Following REE, we postulate a linear equilibrium price

$$\tilde{P} = b_\theta \tilde{\theta} - b_z \tilde{z}, \tag{2.2}$$

where the coefficients b_θ and b_z are to be determined in equilibrium. Informed traders observe both the private signal θ and the price P, thus their optimal portfolio is given by $x_I^*(\theta, P) = \mathbb{E}[\tilde{R}|\theta, P]/(\alpha Var[\tilde{R}|\theta, P])$. Uninformed traders observe only the price, and their optimal portfolio follows $x_U^*(P) = \mathbb{E}[\tilde{R}|P]/(\alpha Var[\tilde{R}|P])$. Next, to determine the coefficients b_θ and b_z we use the market clearing condition,

$$\int_0^1 x_i^* \, di = \lambda x_I^*(\theta, P) + (1-\lambda) x_U^*(P) = \tilde{z}, \tag{2.3}$$

We characterize the financial market equilibrium in the following proposition.

Proposition 2.1. *(Financial Market Equilibrium)*

(i) *The optimal demands of the informed and uninformed are, respectively,*

$$x_I^*(\theta, P) = \frac{\theta - P}{\alpha v_\epsilon}, \qquad x_U^*(P) = \frac{-P}{\alpha v_D (1 + n\lambda/\xi_0)}, \tag{2.4}$$

where

$$\xi_0 = \alpha^2 v_D v_z, \qquad n = \frac{v_\theta}{v_\epsilon}, \qquad v_D = v_\epsilon + v_\theta.$$

(ii) *Given the fraction of informed traders λ, the linear equilibrium price of the risky asset*

$$\tilde{P} = \left(\frac{\lambda \bar{v}}{v_\epsilon}\right) \tilde{\theta} - (\alpha \bar{v}) \tilde{z}, \qquad \frac{1}{\bar{v}} = \frac{\lambda}{v_\epsilon} + \frac{1-\lambda}{v_D \left(1 + \frac{n\lambda}{\xi_0}\right)}. \tag{2.5}$$

Financial market equilibrium is the same as that in the Grossman-Stiglitz model. Here, ξ_0 can be interpreted as an indicator of traders' *risk-sharing incentives.* Intuitively, when risk aversion, payoff risk, and noise-trading risk are high, traders have more incentives to engage in risk-sharing. Moreover, ξ_0 is also related to the Sharpe ratio of the uninformed traders' portfolio in the no-information equilibrium.[7]

The parameter n represents the information-to-noise ratio, which can also be interpreted as a measure of *information quality*.[8] The higher the information quality, the more information benefit informed traders receive from observing the signal $\tilde{\theta}$, as a larger proportion of the payoff uncertainty is resolve by the signal. As in the Grossman-Stiglitz model, we can write down the following ratio between return variances perceived by informed and the uninformed traders,

$$\frac{Var[\tilde{R}|\theta, P]}{Var[\tilde{R}|P]} = 1 - \frac{n\xi_0}{(1+n)(\xi_0 + n\lambda^2)}, \tag{2.6}$$

which is decreasing in n and increasing in λ. Therefore, as the level of informed trading increases, the information advantage informed traders have over the uninformed traders reduces. Again, high information quality increases the benefit for informed traders relative to the uninformed traders.

2.3. **Information Choice and Equilibrium.** For information acquisition, by taking into account the associated cost, trader i makes a probabilistic choice p_i to maximize

$$\mathcal{U}(p_i; \lambda) \equiv [p_i V_I(\lambda) + (1-p_i)V_U(\lambda)]\, e^{\alpha\mu c(p_i)}, \tag{2.7}$$

where $\lambda = \int_0^1 \omega_i \, di$ is a *state variable* representing the *fraction of informed traders* who observe $\tilde{\theta}$, and

$$V_I(\lambda) = \mathbb{E}\left\{\mathbb{E}\left[-e^{-\alpha x_I^*(\theta,P)\tilde{R}}\middle|\theta, P\right]\right\}, \quad V_U(\lambda) = \mathbb{E}\left\{\mathbb{E}\left[-e^{-\alpha x_U^*(P)\tilde{R}}\middle|P\right]\right\}$$

are their maximum expected utilities of the informed and uninformed attainable by the optimal portfolios $x_I^*(\theta, P)$ and $x_U^*(P)$, respectively. Note that $V_I(\lambda)$ and $V_U(\lambda)$ depend on λ since the equilibrium price $\tilde{P}$ itself depends on λ.[9]

[7]In the case of $\lambda = 0$, the risk premium $\mathbb{E}[\tilde{R}|P] = -P = \alpha v_D \tilde{z}$ and the return variance $Var[\tilde{R}|P] = v_D$, therefore the Sharpe ratio can be calculated as $\sqrt{Var\{\mathbb{E}[\tilde{R}|P]\}/Var[\tilde{R}|P]} = \sqrt{\xi_0} = \alpha\sqrt{v_D v_z}$.

[8]In fact, the correlation coefficient $\rho(\tilde{\theta}, \tilde{D})$ between the signal $\tilde{\theta}$ and payoff $\tilde{D}$ satisfies $\rho^2(\tilde{\theta}, \tilde{D}) = n/(1+n)$, therefore the correlation increases in n.

[9]More precisely, in equilibrium, $P_\lambda = h_\lambda(\tilde{\theta}, \tilde{z})$ is a random variable, where h_λ is a deterministic function depending on λ.

Lemma 2.2. *The expected utilities for both informed and uninformed are decreasing in* λ*:* $V_I'(\lambda) < 0, V_U'(\lambda) < 0$*. The same applies to the quantity*

$$\gamma(\lambda) = 1 - \frac{V_I(\lambda)}{V_U(\lambda)}, \tag{2.8}$$

in that $\gamma'(\lambda) < 0$ *for* $\lambda \in [0, 1]$*.*

Note that $\gamma(\lambda)$ measures the potential *utility gain of being informed* relative to being uninformed. This lemma says that the more informed traders in the market, the less incentive to become informed.

Back to utility $\mathcal{U}(p_i; \lambda)$, we assume trader i takes λ as given when choosing his optimal probability p_i^*. More precisely, each trader forms an expectation about the whole vector $(p_j)_{j \in (0,1)}$ and traders' probabilistic choices result in a non-cooperative strategic game. By the first order condition of (2.7), trader i's probabilistic choice p_i^* satisfies

$$\alpha\mu c'(p_i^*) = \frac{\gamma(\lambda)}{1 - p_i^*\gamma(\lambda)}. \tag{2.9}$$

Note that (2.9) can in general have multiple solutions, the following lemma provides a necessary and sufficient condition to guarantee existence and uniqueness of the solution.

Lemma 2.3. *Suppose* $\lambda \in [0, 1]$ *is fixed. A necessary and sufficient condition for* $\mathcal{U}''(p_i; \lambda) < 0$*, is given by*

$$\gamma(\lambda) \leq \min_p \left[\frac{K(p)}{2 + K(p)p} \right], \tag{2.10}$$

where $K(p) = (\alpha\mu c'(p) + c''(p)/c'(p))$*. In case* $c(p) = p^2$*, (2.10) can be written as*

$$\gamma(\lambda) \leq \frac{2\alpha\mu + 1}{2\alpha\mu + 3}. \tag{2.11}$$

Based on Lemma 2.3, taking the level of informed trading, λ, as given, in order for the optimization problem to be well-defined, the relative utility gain of being informed cannot be too large. Intuitively, traders pay more to increase their potential information benefit. Note that the marginal cost of information acquisition increases, while the marginal benefit of being informed decreases. Therefore, when the potential information benefit is relatively low, it would dominate the cost and hence it is optimal to acquire informaiton. However, this can become the opposite when the potential information benefit is too large. It is helpful to look at two special cases, namely $\mu \to \infty$ and $\mu \to 0$. In the first case, the condition becomes $\gamma(\lambda) \leq 1$, which is always satisfied. Intuitively, when cost sensitivity is extremely high, traders' information choice converges to $p^* = 0$, thus the equilibrium level of informed trading $\lambda = 0$. In the second case, the condition becomes $\gamma(\lambda) \leq 1/3$, which is not always satisfied. Therefore, it is possible that $\mathcal{U}(p_i, \lambda)$ exhibits more than one local maxima.[10]

Note that the optimization scheme of the joint information and portfolio choice for trader i can be separated in two stages and solved using backward induction. At $t = 1$, trader i's type is revealed. Given his type, his portfolio choice x_i^* can be determined and hence the value functions, $V_I(\lambda)$ and $V_U(\lambda)$, can be computed. At $t = 0$, traders play an information game. By averaging the likelihood of being informed and forming an expectation about other traders' actions, traders strategically choose optimal strategies, $(p_i^*)_{i\in(0,1)}$. Finally, to close the model, we require $\lambda = \int_0^1 p_i^* \, di$, *i.e.*, the market fraction of informed traders must be consistent with traders' strategic probabilistic choices in a Nash equilibrium. Formally, we introduce the following definition of equilibrium for information choice.

Definition 2.1. *The probabilities $p^* = (p_i^*)_{i\in(0,1)}$ and (expected) market fraction of informed traders λ are in equilibrium if*

(i) $p^* = (p_i^*)_{i\in(0,1)}$ *is a Nash equilibrium, meaning that for every $i \in (0,1)$,*[11]

$$\mathcal{U}(p_i^*; \lambda) \geq \mathcal{U}(p_i; \lambda) \qquad \textit{for all } p_i \in [0,1];$$

(ii) *the following* consistency condition *is satisfied*[12]

$$\lambda = \mathbb{E}\left[\int_0^1 \omega_i^* \, di\right] = \int_0^1 p_i^* \, di, \tag{2.12}$$

[10]For example, for $n = 6$, and by fixing parameters of the model at level $\xi_0 = 1$, $v_\epsilon = 0.1$, $\alpha = 0.5$ and $\mu = 1.574$, the value of $\mathcal{U}$ reaches two local maxima at $p = 0.7$ and at $p = 1$. Moreover, the expected utility at the two local maxima is approximately -0.5871 for both of them.

[11]With a slight abuse of notation, we write $\mathcal{U}(p_i; \lambda)$ in place of $\mathcal{U}(p_i; p_{-i}^*)$, where $p_{-i}^* = (p_j^*)_{j\neq i}$. Indeed, the only payoff-relevant variable for the information game is λ; moreover, having a continuum of traders, the contribution of trader i on the realization of λ is negligible.

[12]At the equilibrium, the expectations are realized so that the fraction of informed, λ, exactly matches the value expected by the traders when using the revealed vector of probabilities p^*.

here ω_i^* is the random variable associated with the optimal probability p_i^*;

We now characterize the equilibrium fraction of informed traders and market price and their implications. For clarity, we summarize in one unique modeling assumption the few conditions ensuring the existence of a unique equilibrium λ, monotonically decreasing in the cost sensitivity parameter μ, in case the cost $c(p)$ is quadratic. These will be the conditions we will assume when analysing welfare implications in the remainder of this article.

Assumption 2.2. *The cost for information is quadratic, namely, $\mu c(p) = \mu\, p^2$. Moreover, the following conditions on parameters hold true:*

$$n \leq 3; \quad \mu > \bar{\mu} \equiv \frac{1}{2\alpha}\frac{\gamma(1)}{1-\gamma(1)} = \frac{1}{2\alpha}\left(\sqrt{\frac{n(n+1)+2\xi_0}{n(n+1)+\xi_0}} - 1\right),$$

The first condition, $n \leq 3$, guarantees that trader's optimization problem is well-defined for any level of informed trading, i.e., λ, in equilibrium. Note that a value of n between 2 and 3 is consistent with real financial markets. Brogaard, Nguyen, Putniņš and Wu (2020) reports a 31% noise to total variance ratio for equity markets, corresponding to $n \approx 2.23$. The second condition, together with $n \leq 3$, guarantees the existence of a unique equilibrium $0 \leq \lambda < 1$, where λ decreases in μ.[13]

Proposition 2.4. *(Information Choice Equilibrium)*
Under Assumption 2.2, the level of informed trading, λ, satisfies

$$\alpha\mu\, c'(\lambda) = \frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}, \tag{2.13}$$

where λ is unique and decreasing in μ, and

$$\gamma(\lambda) \leq \frac{1}{2}. \tag{2.14}$$

[13]For more details, see Appendix B. Note that, when $\mu \leq \bar{\mu}$, λ is fixed at 1. In this case, the welfare can be improved as the cost decreases further (as in Grossman-Stiglitz model). However, we are interested in whether the decrease in the cost and hence the increase in informed trading can improve the welfare. Moreover, we know that the full-information equilibrium $\lambda = 1$ is Pareto-inefficient, since it is always dominated by the no-information equilibrium $\lambda = 0$, even if information is costless ($\mu = 0$). Therefore, we do not consider the corner equilibrium $\lambda = 1$ in this paper.

Several features of Proposition 2.4 deserve comments. First, while the financial market equilibrium is identical to that in the Grossman-Stiglitz model, the information acquisition decision equilibrates differently. In the absence of information acquisition uncertainty, the Grossman-Stiglitz model requires $V_I(\lambda)e^{\alpha c} = V_U(\lambda)$ where $c > 0$ is a fixed cost. Thus, informed trading increases as information acquisition cost c decreases. In the probabilistic-choice model, due to *ex-ante* homogeneity of investors, every trader makes the *same* optimal probabilistic choice $p_i^* = p^*(\lambda) = \lambda$, which satisfies (2.13). Intuitively, the optimal probability choice p^* and hence level of informed trading λ increase when the cost sensitivity μ decreases.[14]

Second, market exhibits *strategic substitutability* in the probabilistic choices. Assume a quadratic cost function, solving the first order condition (2.9) gives the following optimal probability,

$$p^*(\lambda) = \frac{\alpha\mu - \sqrt{\alpha^2\mu^2 - 2\alpha\mu\gamma^2(\lambda)}}{2\alpha\mu\gamma(\lambda)}, \tag{2.15}$$

which equals to λ after we substitute $\alpha\mu = \gamma(\lambda)/(2\lambda(1-\lambda\gamma(\lambda)))$.[15] It can be shown from (2.15) that $dp^*(\lambda)/d\lambda = (\partial p^*/\partial\gamma)\gamma'(\lambda) < 0$, which implies that more informed trading *reduces* the incentives for traders to increase their optimal probability choice. Furthermore, note that market efficiency can be measured by the correlation coefficient $\rho_\theta = \mathbb{C}orr(\tilde{P}, \tilde{\theta})$, which based on the equilibrium price in (2.5) can be written as

$$\rho_\theta(\lambda)^2 = \frac{1}{1+m(\lambda)}, \qquad m(\lambda) = \frac{\xi_0}{\lambda^2 n(1+n)}. \tag{2.16}$$

Therefore, for given information quality and risk-sharing incentives, higher market efficiency *reduces* the potential utility gain of being informed, since $\gamma(\lambda) = 1 - 1/\sqrt{1+n(1-\rho_\theta(\lambda)^2)}$ according to (2.9).

Third, based on (2.16), more informed trading improves market efficiency but, since $V_I'(\lambda) < 0$ and $V_U'(\lambda) < 0$ (as seen in Lemma 2.2), is detrimental to the

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[14]This holds true in particular under Assumption 2.2. We provide more general sufficient conditions for the uniqueness in Appendix B. In principle, there could be multiple equilibria in λ for the fixed point argument (2.13) even if the optimization problem is well-defined in p^*. We leave this intriguing discussion on multiple equilibria for future research.

[15]There is a second solution $p^* = (\alpha\mu + \sqrt{\alpha^2\mu^2 - 2\alpha\mu\gamma^2(\lambda)})/(2\alpha\mu\gamma(\lambda))$, which can be ruled out under Assumption 2.2.

expected utilities of informed and uninformed traders. This may seem counter-intuitive, since more informed trading helps to resolve payoff uncertainty which should improve expected utility for traders. However, as Kurlat and Veldkamp (2015) explain, "*decreasing risk lowers the equilibrium return and systematically raises the assets average price. For welfare, this means that information reduces the assets risk, but also implies lower return. With exponential utility and normally distributed payoffs, the return effect always dominates.*" Interestingly, this *return effect* can be traced back to the *residual risk* faced by traders. In fact, traders' squared expected utilities can be written as[16]

$$V_I^2(\lambda) = \frac{Var[\tilde{R}|\theta, P]}{Var[\tilde{R}]}, \quad V_U^2(\lambda) = \frac{Var[\tilde{R}|P]}{Var[\tilde{R}]}. \tag{2.17}$$

Since $V_K'(\lambda) < 0$, $K = I, U$, from (2.17) we can conclude that, because $dV_K^2(\lambda)/d\lambda > 0$, more informed trading actually increases the residual risk faced by traders, i.e., it increases the conditional return variance as a proportion of the total return variance, hence is detrimental to traders' expected utilities.

In the Grossman and Stiglitz model, since welfare is measured by $V_U(\lambda)$, informed trading is always welfare reducing, hence information acquisition and more efficient markets are not socially desirable (with respect to welfare). However, with information acquisition uncertainty, this is not necessarily true for traders' *ex-ante* welfare. As we show in Section 3, information acquisition can be welfare improving under certain market conditions.

3. Welfare Analysis

This section examines the *ex-ante* welfare of the utility-maximizing traders in the noise-trader model in Section 2. The welfare can be measured as a function of the state variable λ only by virtue of the first order condition (2.13),

$$\mathcal{W}(\lambda) \equiv \mathcal{U}(p^*, \lambda) = \bar{V}(p^*, \lambda) e^{\Phi(p^*, \lambda)}, \qquad p^* \equiv p^*(\lambda) = \lambda, \tag{3.1}$$

[16]Note that, for $K = I, U$, $V_K(\lambda) = -1/\sqrt{1 + \xi_K(\lambda)}$ and $\xi_K(\lambda) = Var\{\mathbb{E}[\tilde{R}|\mathcal{F}_K]\}/Var[\tilde{R}|\mathcal{F}_K]$. Therefore, $V_K^2(\lambda) = Var[\tilde{R}|\mathcal{F}_K]/(Var\{\mathbb{E}[\tilde{R}|\mathcal{F}_K]\} + Var[\tilde{R}|\mathcal{F}_K])$. Then (2.17) follows from the law of total variance.

where

$$\bar{V}(p^*, \lambda) = p^* V_I(\lambda) + (1 - p^*) V_U(\lambda), \qquad \Phi(p^*, \lambda) = \frac{c(p^*)}{c'(p^*)} \frac{\gamma(\lambda)}{1 - p^*\gamma(\lambda)}.$$

The marginal welfare can be separated into two components,

$$\begin{aligned} \frac{\mathcal{W}'(\lambda)}{-\mathcal{W}(\lambda)} &= \frac{1}{-\mathcal{U}} \frac{\partial \mathcal{U}}{\partial p^*} + \frac{1}{-\mathcal{U}} \frac{\partial \mathcal{U}}{\partial \lambda} \\ &= \underbrace{\frac{\gamma(\lambda)}{1 - \lambda\gamma(\lambda)} - \frac{\partial \Phi}{\partial p^*}}_{\text{information-gain effect}} + \underbrace{\frac{\lambda V_I'(\lambda) + (1-\lambda) V_U'(\lambda)}{-\bar{V}(p^*, \lambda)} - \frac{\partial \Phi}{\partial \lambda}}_{\text{informed-trading effect}}. \end{aligned} \tag{3.2}$$

This decomposition disentangles the effect of an increase in the optimal probability choice, i.e., an increase in p^* while holding λ constant, from an increase in the level of informed trading, i.e., an increase in λ while holding p^* constant. We refer the first as the *information-gain effect* and the second as the *informed-trading effect*. To better understand their joint effect, we first examine each effect separately. For tractability, from now on we assume that Assumption 2.2 holds true. This allows us to give a sharper economic intuition about the marginal contributions of the two effects on the marginal welfare.

3.1. **Information-Gain Effect.** This effect characterizes a trade-off between the relative benefit of being informed, $\gamma(\lambda)/(1-\lambda\gamma(\lambda))$, and the cost, $\partial\Phi(p^*, \lambda)/\partial p^*|_{p^*=\lambda}$, due to information acquisition in equilibrium. Note that, for $c(p) = p^2$, the marginal cost of the information acquisition is always positive, i.e.,

$$\left. \frac{\partial \Phi(p^*, \lambda)}{\partial p^*} \right|_{p^*=\lambda} = \frac{1}{2} \frac{\gamma(\lambda)}{(1 - \lambda\gamma(\lambda))^2} > 0.$$

Therefore, the information-gain effect reflects the net effect of the trade-off,

$$\frac{1}{-\mathcal{U}} \frac{\partial \mathcal{U}}{\partial p^*} = \frac{\gamma(\lambda)}{1 - \lambda\gamma(\lambda)} - \frac{\partial \Phi}{\partial p^*} = \frac{\gamma(\lambda)(1 - 2\lambda\gamma(\lambda))}{2(1 - \lambda\gamma(\lambda))^2},$$

which is positive if and only if $\lambda\gamma(\lambda) < 1/2$. Note that the first component in the information-gain effect, marginal benefit of an increase in p^*, is decreasing in λ while the second component, marginal cost of an increase in p^*, is increasing in λ. Hence, we require either λ or $\gamma(\lambda)$ to be low for the overall effect to be positive. Under Assumption 2.2, $\gamma(\lambda) < 1/2$, therefore, the information-gain effect is strictly positive. Interestingly, a small $\gamma(\lambda)$ can ensure traders' optimization problem is well-defined and also that welfare in equilibrium improves when traders' optimal probabilistic information choice increases. The same intuition for $\gamma(\lambda)$ being small applied to Lemma 2.3 also applies here.

Moreover, note that the information-gain effect is non-existent in the Grossman-Stiglitz model, since in their equilibrium, informed and uninformed traders are assumed to have the same expected utility. In contrast, in the probabilistic-choice model, information-gain effect comes from traders' anticipation of potentially becoming informed next period, and is strictly positive in equilibrium under Assumption 2.2.

3.2. **Informed-Trading Effect.** This effect characterizes the traditional trade-off between the informed trading, $(\lambda V_I'(\lambda)+(1-\lambda)V_U'(\lambda))/(-\bar{V}(\lambda,\lambda))$, and the marginal cost of informed trading, $\partial\Phi(p^*,\lambda)/\partial\lambda$, in the Grossman-Stiglitz model. Note that

$$\frac{\lambda V_I'(\lambda)+(1-\lambda)V_U'(\lambda)}{-\bar{V}(\lambda,\lambda)} = -\left[\Gamma_1(\lambda)\frac{V_I'}{V_I}+(1-\Gamma_1(\lambda))\frac{V_U'}{V_U}\right],$$

where $\Gamma_1(\lambda) = \lambda(1-\gamma(\lambda))/(1-\lambda\gamma(\lambda)) > 0$. Based on the discussion in Section 2, due to the fact that $V_K'(\lambda)/V_K(\lambda) = (1/2)(dV_K^2(\lambda)/d\lambda)/V_K^2(\lambda) > 0$ for $K = I, U$, the informed trading increases the residual risk faced by traders and therefore contributes to the marginal welfare negatively. Also, note that

$$\left.\frac{\partial\Phi(p^*,\lambda)}{\partial\lambda}\right|_{p^*=\lambda} = \frac{1}{2}\frac{\lambda\gamma'(\lambda)}{(1-\lambda\gamma(\lambda))^2}.$$

Since $\gamma'(\lambda) < 0$, the marginal cost of informed trading is always negative (as a cost saving in the Grossman-Stiglitz model), which contributes to the marginal welfare positively. The informed-trading effect reflects the net effect of this trade-off,

$$\frac{1}{-\mathcal{U}}\frac{\partial\mathcal{U}}{\partial\lambda} = -\left[\Gamma(\lambda)\frac{V_I'}{V_I}+(1-\Gamma(\lambda))\frac{V_U'}{V_U}\right],$$

where $\Gamma(\lambda) = \frac{\lambda(1-2\lambda\gamma)(1-\gamma)}{2(1-\lambda\gamma)^2}$. Therefore, under Assumption 2.2, the informed-trading effect is negative. This means that a higher level of informed trading can be detrimental to welfare (as in the Grossman-Stiglitz model). Intuitively, more informed trading increases the residual risk faced by traders and hence decreases their expected utilities. Although there is a cost saving, the overall welfare effect of informed trading remains negative.

3.3. **Marginal Welfare.** Based on the above analysis, we see that the two effects work in opposite direction, and their trade-off determines the marginal welfare.

Proposition 3.1. *(Marginal Welfare) Under Assumption 2.2, marginal welfare is positive (i.e.,* $\mathcal{W}'(\lambda) > 0$*) if and only if*

$$\frac{\gamma(\lambda)(1 - 2\lambda\gamma(\lambda))}{2(1 - \lambda\gamma(\lambda))^2} > \Gamma(\lambda)\frac{V_I'(\lambda)}{V_I(\lambda)} + (1 - \Gamma(\lambda))\frac{V_U'(\lambda)}{V_U(\lambda)}. \tag{3.3}$$

Proposition 3.1 shows that information acquisition, albeit costly, can be beneficial to welfare. Particularly, a positive marginal welfare requires the information-gain effect to dominate the informed-trading effect. To explore market condition for such dominance, we first examine a special case of the welfare improvement at no-information equilibrium. Applying Proposition 3.1 to the case of $\lambda = 0$, we have the following result.

Corollary 3.1. *At the no-information equilibrium,*

(i) *marginal welfare is positive (i.e.,* $\mathcal{W}'(0) > 0$*) if and only if*

$$\underbrace{\frac{1}{2}\left(1 - \frac{1}{\sqrt{1+n}}\right)}_{\text{information-gain effect}} \quad + \quad \underbrace{\frac{-n\xi_0}{1+\xi_0}}_{\text{informed trading effect}} > 0; \tag{3.4}$$

(ii) *if marginal welfare is positive (i.e.,* $\mathcal{W}'(0) > 0$*), then* $\xi_0 < 1/3$.

At the no-information equilibrium, Corollary 3.1 (i) provides an explicit expression for the information-gain and informed-trading effects with respect to information quality (n) and risk-sharing incentives (ξ_0). We have two observations about the necessary and sufficient condition (3.4). First, the informed-trading effect is stronger when risk-sharing incentives are high, hence informed trading brings more distortion to risk-sharing. Second, both informed-trading and information-gain effects are stronger when information quality is high. Intuitively, a more precise signal helps informed traders to resolve more payoff uncertainty. On the one hand, it gives informed traders more advantage over the uninformed traders. On the other hand, it also leads to a worse return effect. To see the net effect of n on the marginal welfare, we rewrite condition (3.4) as

$$\frac{\xi_0}{1+\xi_0} < \frac{1}{2n}\left(1 - \frac{1}{\sqrt{1+n}}\right).$$

Since the function on the right hand side decreases in n, the net effect of n is negative. Therefore, we conclude from these two observations that the effects of both risk-sharing incentives and information quality on the marginal welfare at no-information equilibrium are unambiguously negative. This implies that welfare improvement at the no-information equilibrium is more likely in an economy with both low risk-sharing incentives and information quality.

Corollary 3.1 (ii) shows that when $\xi_0 > 1/3$, the negative informed-trading effect always dominates at the no-information equilibrium. Intuitively, when risk-sharing incentives are sufficiently strong, risk-sharing distortion due to informed trading dominates marginal welfare, as a result, information acquisition is always welfare reducing and the most preferable situation (from a welfare viewpoint) is the no-information equilibrium.

The above intuitions on the roles of information quality and risk-sharing incentives also apply to the welfare improvement in general asymmetry information equilibrium $\lambda > 0$. Under Assumption 2.2, it can be shown that

$$\frac{d}{d\lambda}\left(\frac{1}{-\mathcal{U}}\frac{\partial \mathcal{U}}{\partial p^*}\right) = -\frac{2\lambda\gamma(\lambda)^3 - \gamma'(\lambda)(1-3\lambda\gamma(\lambda))}{2(1-\lambda\gamma(\lambda))^3} < 0. \tag{3.5}$$

This implies that the information-gain effect is always decreasing in informed trading λ. The behavior of the informed trading effect with respect to λ is however less trivial. At the no-information equilibrium, assuming $\xi_0 < 1/3$, the welfare cost of informed trading satisfies

$$\left.\frac{d}{d\lambda}\left(\frac{1}{\mathcal{U}}\frac{\partial \mathcal{U}}{\partial \lambda}\right)\right|_{\lambda=0} = \frac{n}{1+\xi_0} + \left(\frac{n}{1+\xi_0}\right)^2\left[(1-\xi_0^2) + (1-\xi_0)\right] > 0. \tag{3.6}$$

Therefore, with more informed trading, the information-gain effect weakens while the informed trading effect strengthens. After informed trading reaches a certain level, the negative informed trading effect prevails.

From Corollary 3.1, we have the dominance of the information-gain effect at $\lambda = 0$ under condition (3.4). Since the information-gain effect is decreasing for $\lambda \in [0, 1)$, while the informed-trading effect is increasing in λ for small λ, there exists a $\lambda^* > 0$ at which $\mathcal{W}'(\lambda^*) = 0$, From Proposition 2.4, this implies that, corresponding to λ^*, there is a unique cost sensitivity $\mu^* = \mu(\lambda^*)$ such that marginal welfare $\mathcal{W}'(\lambda)$ is positive for $\mu > \mu^*$. We call λ^* the *Pareto-optimal state* for the utility-maximizing traders (speculators) who provide liquidity to noise traders. We now provide a sufficient condition for the existence of a unique Pareto-optimal state.

Corollary 3.2. *Under Assumption 2.2 and for* $\xi_0 < 1/11$*, there exists an optimal cost sensititivty* $\mu^* > 0$ *that corresponds to the Pareto optimal state* $\lambda^* > 0$ *such that marginal welfare* $\mathcal{W}'(\lambda) > 0$ *for* $\lambda \in [0, \lambda^*)$ *and* $\mu > \mu^*$*.*

In Proposition 2.4, we have seen that the unique equilibrium informed trading λ decreases in μ. Corollary 3.2 further confirms that the welfare improvement is more likely to occur at higher level of cost sensitivity, $\mu > \mu^*$, or correspondingly at lower level of informed trading $\lambda \in [0, \lambda^*)$. The sufficient condition further confirms that welfare improvement is more likely in an economy with both low risk-sharing incentives (ξ_0) and information quality (n), achieving Pareto optimality at λ^*. Note that the lower bound for ξ_0 corresponds to a Sharpe ratio of 0.3 in the no-information equilibrium, below this level any information quality, $n < 3$, can potential improve welfare for small enough λ. Intuitively, when traders have relatively less incentives to trade for risk-sharing purposes, the welfare cost of informed trading is also relatively small, hence it is easier for the anticipatory welfare benefit of probabilistic information choice to dominate traders' marginal welfare.

3.4. **Numerical Analysis.** We now perform a numerical analysis to verify our results and more importantly examine the market conditions for welfare improvement with respect to informed trading (λ), information quality (n) and risk-sharing incentive (ξ_0). Results are presented in Figure 3.1, from which we have the following observations.

Firstly, information acquisition is *less* likely to improve the welfare in markets with relatively high information quality (n), risk-sharing incentives (ξ_0), and informed

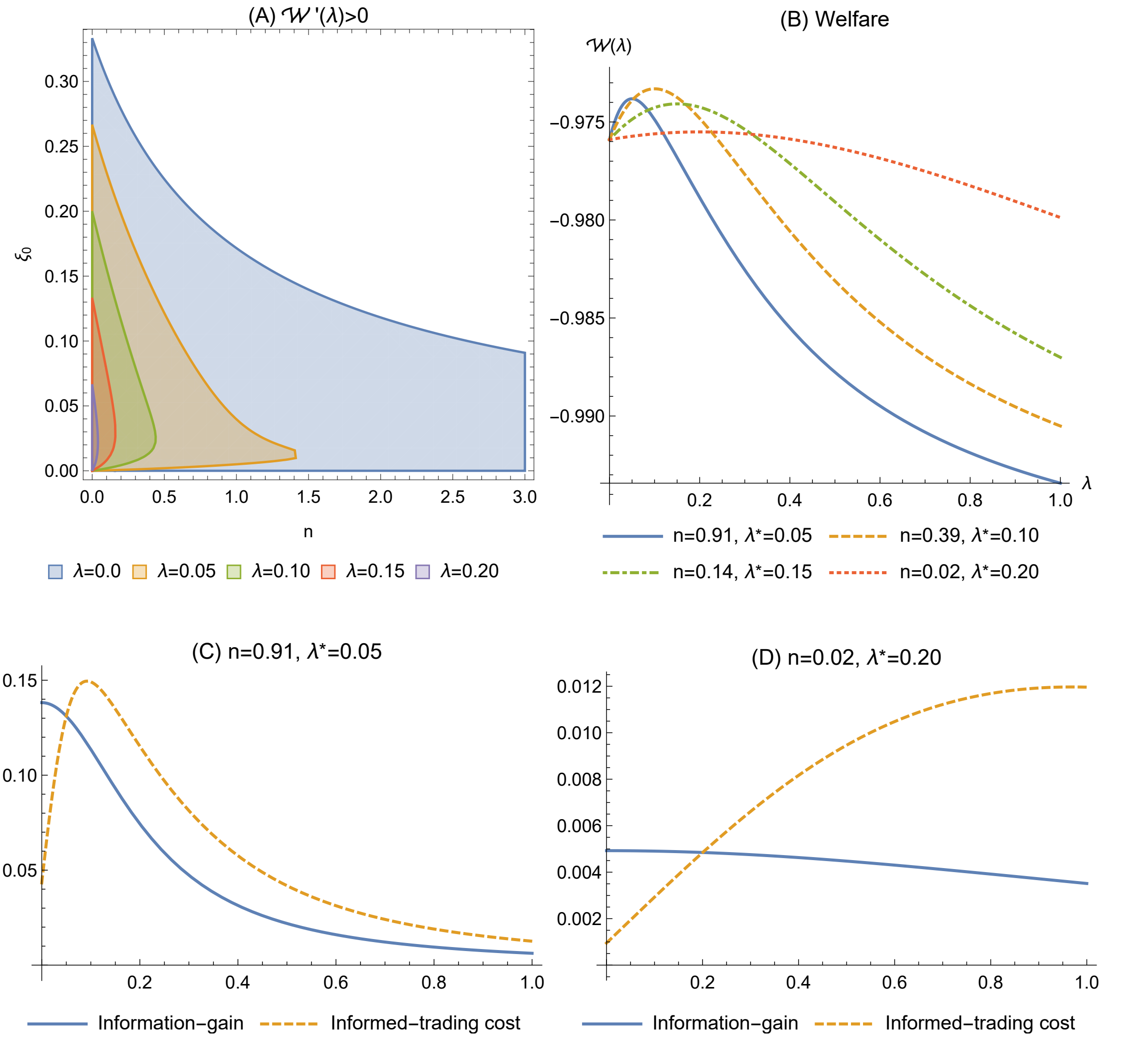

FIGURE 3.1. Panel (A) shows the parameter region of (n, ξ_0) in which $\mathcal{W}'(\lambda) > 0$ for a given λ. Panel (B) shows $\mathcal{W}(\lambda)$ for $0 \leq \lambda \leq 1$, where $\xi_0 = 0.05$ and n is chosen in such a way that $\mathcal{W}'(\lambda^*) = 0$. Panels (C) and (D) show information-gain benefit versus informed-trading cost, where $\xi_0 = 0.05$ and $n = 0.91$ in (C) and $n = 0.02$ in (D).

trading level (λ). This is shown in Panel (A) that the parameter region of (n, ξ_0) for $\mathcal{W}'(\lambda)$ shrinks in both n and ξ_0. This observation is consistent with the analytical results obtained above.

Secondly, the region for $\mathcal{W}'(\lambda) > 0$ shrinks in λ, and we provide the following explanation. As the level of informed trading rises, there is less utility to be gained by being informed, this is illustrated in both Panels (C) and (D).[17] They show that

[17]To illustrate the trade-off between the two effects, since the informed trading effect is negative, we plot $(1/\mathcal{U})\partial\mathcal{U}/\partial\lambda$, as the welfare cost of informed trading in Panels (C) and (D).

the information-gain effect (the solid lines) is decreasing in λ, while the informed-trading effect (the dotted lines) is initially increasing and then decreasing in λ when λ is high enough and information quality is relatively high.

Thirdly, for given parameters n and ξ_0, the welfare function is hump-shaped, reaching the Pareto-optimal state at λ^* at which $\mathcal{W}'(\lambda^*) = 0$, as illustrated in Panel (B). Since high information acquisition cost is associated with larger information gain, one could interpret the cost sensitivity, μ as a tax on information acquisition. Therefore, when a policymaker is able to tighten or loosen the restrictions on financial market regulations, our results show that a decrease in the information tax rate, from μ to μ^*, can result in an welfare improvement in informed trading at the level of $\lambda \in [0, \lambda^*)$.

Based on this observation, we next provide some welfare implications by focusing on the dependence of the Pareto-optimal state λ^* (and hence the optimal cost sensitivity μ^*) on the information quality (n) and risk-sharing incentive (ξ_0), as illustrated in Figure 3.1(B).

3.5. **Welfare Implications.** With market characteristics exogenously specified with respect to information quality (n) and risk-sharing incentives (ξ_0), we first examine how a policymaker should adjust cost sensitivity μ (and hence the informed trading level λ) in order to maximize trader' welfare. We then examine how such policy adjustment changes with information quality (n) and risk-sharing incentive (ξ_0).

Based on the hump-shaped welfare function, we conduct an analysis on the Pareto-optimal welfare achieved. In Figure 3.2, for three different values of ξ_0 and $n < 3$, we report: (A) the Pareto-optimal state λ^*; (B) the optimal cost sensitivity μ^*; (C) welfare at the Pareto-optimal state relative to that at the no-information equilibrium; and (D) the corresponding information efficiency $\rho_\theta(\lambda^*)$. The results provide several welfare implications.

Firstly, Panels (A) and (B) show that, to maximize welfare, a policymaker should choose a higher cost sensitivity to discourage information acquisition when both information quality and risk-sharing incentives are high. This is consistent with the literature (e.g., Allen, 1984). Intuitively, with high information quality, traders

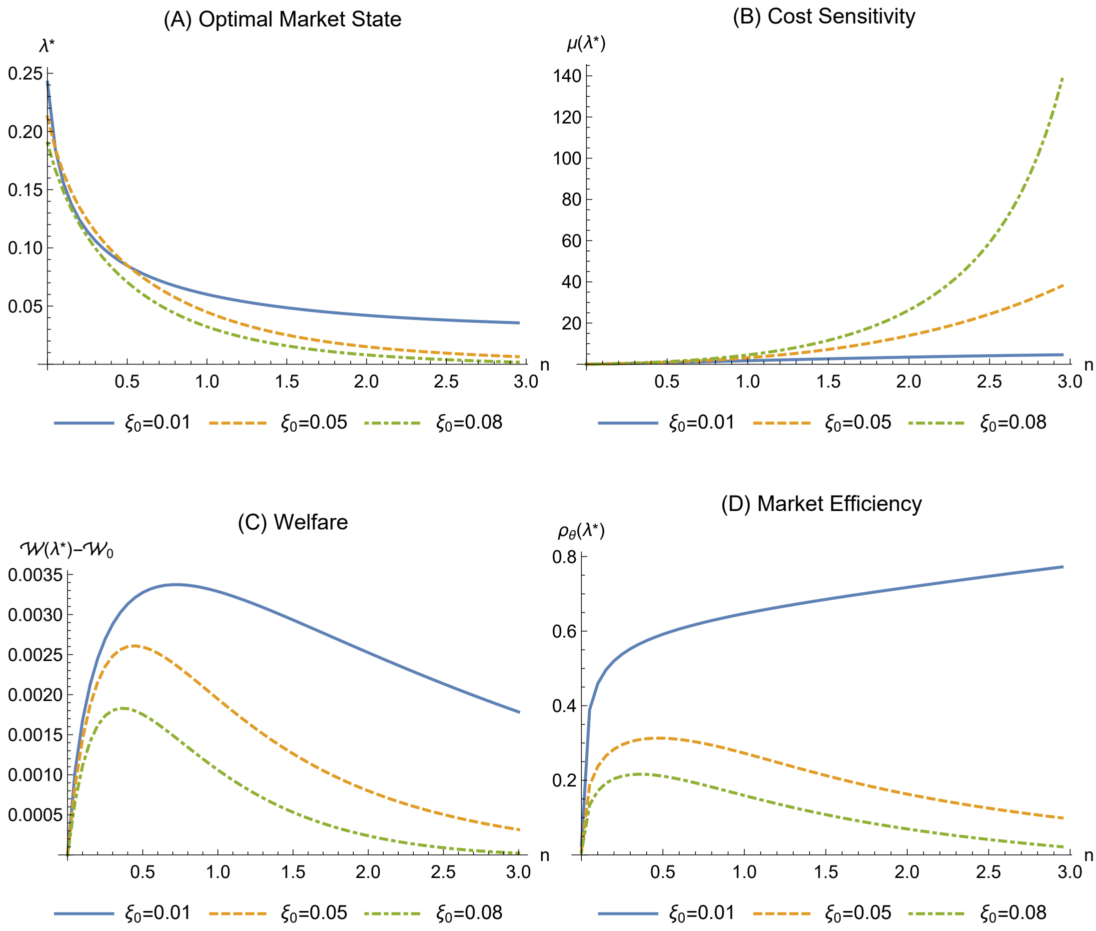

FIGURE 3.2. The relationship between information quality n and Pareto-optimal state λ^* in Panel (A), cost sensitivity $\mu(\lambda^*)$ in Panel (B), welfare $\mathcal{W}(\lambda^*)$ in Panel (C) and market efficiency $\rho_\theta(\lambda^*)$ in Panel (D), here $\mathcal{W}_0 = -1/\sqrt{1+\xi_0}$ is the welfare in the no-information equilibrium.

have more incentives to acquire information, which increases the distortion of risk-sharing, in particular, when investors' risk-sharing incentives are high. Therefore, to improve welfare the policymaker can make information acquisition more costly, which reduces informed trading and thus the distortion of risk-sharing. Note that the Pareto-optimal state λ^* decreases monotonically with information quality, but not always in risk-sharing incentives. Moreover, when risk-sharing incentives are low, λ^* decreases at a slower rate.

Secondly, Panel (C) shows that the Pareto-optimal welfare decreases in the risk-sharing incentives but is concave in information quality, in particular when risk-sharing incentives are high. As we have discussed, high risk-sharing incentives increase the distortion of risk-sharing and hence reduce welfare. The welfare improvement from the no-information equilibrium reaches its maximum at an intermediate level of information quality, the improvement is greater for lower risk-sharing incentives. The welfare benefit of information acquisition can be substantial, especially when risk-sharing incentives are low. Table 1 shows that when $\xi_0 = 0.01$, i.e., the Sharpe ratio at the no-information equilibrium is $\sqrt{\xi_0} = 0.1$, and the risk-sharing benefit $1 + \mathcal{W}_0 \approx 0.05\%$. In this case, having an informed trading level of $\lambda^* = 0.071$ by a cost sensitivity of $\mu = 1.195$ can lead to a welfare improvement of $\Delta\mathcal{W} = 0.0374\%$ (when $n = 0.72$), which is a 68% increase. This (proportional) welfare improvement becomes 120% when $\xi_0 = 0.005$ and $\lambda^* = 0.054$, and 351% when $\xi_0 = 0.001$ and $\lambda^* = 0.026$.

	$\xi_0 = 0.01$	$\xi_0 = 0.005$	$\xi_0 = 0.001$
$1 + \mathcal{W}_0$	4.963	2.491	0.500
$\Delta\mathcal{W}$	3.374	2.990	1.757
$\Delta\mathcal{W}/(1 + \mathcal{W}_0)$	**0.680**	**1.200**	**3.514**
n	0.72	0.84	1.03
λ^*	0.071	0.054	0.026
$\mu(\lambda^*)$	1.195	1.538	3.164

TABLE 1. Welfare benefit of risk-sharing $1 + \mathcal{W}_0$ (in basis points) at the no-information equilibrium, and welfare improvement $\Delta\mathcal{W} \equiv \mathcal{W}(\lambda^*) - \mathcal{W}_0$ (in basis points), relative improvement $\Delta\mathcal{W}/(1 + \mathcal{W}_0)$, information quality n, level of informed trading λ^* and cost sensitivity $\mu(\lambda^*)$ at the Pareto-optimal state.

Moreover, together with Panel (A), the Pareto-optimal welfare reflects a trade-off between the information quality (n) and quantity (λ^*). The concave Pareto-optimal welfare in Panel (C) also indicates that a globally maximum welfare can be achieved with relative high information quality and cost when risk-sharing incentive is low, but with relative low information quality and cost when risk-sharing incentive is high.

Thirdly, Panels (C) and (D) show that market efficiency is not necessarily in conflict with welfare, they can both improve with higher information quality provided that the market is at the Pareto-optimal state.

Overall, our results show that, to maximize traders' welfare, policymakers should increase cost sensitivity (μ^*) in response to high information quality, in order to drive down informed trading to the Pareto-optimal state (λ^*), however, not necessarily to zero (no-information equilibrium). Moreover, with intermediate level of information quality and low risk-sharing incentives, the relative welfare improvement from the no-information equilibrium can be substantial. In other words, there can be a large *welfare loss* for the liquidity providers in the market if the opportunity for information acquisition was completely absent.

4. Modelling Trading Motives Explicitly

In this section, rather than assuming exogenous noise in supply, we follow Manzano and Vives (2011) and Bond and Garcia (2018) to motivate trading by assuming that traders receive private *endowment shocks*.

Each trader i receives an endowment, $e_i\tilde{D}$, at the end of the trading period. Thus, trader i's future wealth is given by

$$\tilde{W}_i = (x_i + e_i)\tilde{R} + e_i\tilde{P} - \mu\, c(p_i), \qquad \tilde{R} = \tilde{D} - \tilde{P}. \tag{4.1}$$

We assume e_i is known to trader i, whereas other traders only have common knowledge about the distribution function from which e_i is drawn. In particular, $e_i = \tilde{z} + \tilde{u}_i$, where $\tilde{z} \sim \mathcal{N}(0, v_z)$ is an aggregate endowment shock and $\tilde{u}_i \sim \mathcal{N}(0, v_u)$ is an idiosyncratic shock, thus $v_e \equiv \mathbb{V}ar[\tilde{e}_i] = v_z + v_u$.

4.1. **Optimization problem.** As before, trader i's objective is to determine the optimal probability p_i^* of observing the private signal θ, in order to maximize his expected utility of terminal wealth,

$$\mathcal{U}(p_i; \lambda, e_i) \equiv [p_i V_I(\lambda, e_i) + (1 - p_i)V_U(\lambda, e_i)]\, e^{\alpha\mu\, c(p_i)}, \tag{4.2}$$

where

$$V_I(\lambda, e_i) = \mathbb{E}\left\{\mathbb{E}\left[-e^{-\alpha\left(x_I^*(\theta,P,e_i)\tilde{R}+e_i\tilde{D}\right)}\middle|\theta, P, e_i\right]\middle|e_i\right\}$$

and

$$V_U(\lambda, e_i) = \mathbb{E}\left\{\mathbb{E}\left[-e^{-\alpha\left(x_U^*(P,e_i)\tilde{R}+e_i\tilde{D}\right)}\middle|P, e_i\right]\middle|e_i\right\}$$

are trader i's expected utility, depending on whether or not he observes the private signal θ. Therefore, beside θ, trader i also has a different private signal about his own endowment shock e_i. Intuitively, e_i helps trader i to forecast the aggregate endowment shock $\tilde{z}$, which is negatively correlated with the equilibrium price $\tilde{P}$. For example, after observing the same price, a trader who receives a positive endowment shock will infer a larger value for θ than a trader who receives a negative endowment shock.

Conditional on his information set, trader i's optimal portfolio is given by

$$x_i^* = \begin{cases} x_I^*(\theta, P, e_i) = \dfrac{\mathbb{E}[\tilde{R}|\theta, P, e_i]}{\alpha \mathbb{V}ar[\tilde{R}|\theta, P, e_i]} - e_i, & \mathcal{F}_i = \{\theta, P, e_i\}; \\ x_U^*(P, e_i) = \dfrac{\mathbb{E}[\tilde{R}|P, e_i]}{\alpha \mathbb{V}ar[\tilde{R}|P, e_i]} - e_i, & \mathcal{F}_i = \{P, e_i\}. \end{cases} \quad (4.3)$$

As before, we conjecture a linear equilibrium price,

$$\tilde{P} = b_\theta \tilde{\theta} - b_z \tilde{z}. \quad (4.4)$$

Next, we characterize the solution to traders' optimization problem. The optimal demands for the uninformed and informed traders are given by

$$x_U^*(P, e_i) = \frac{-(1-\kappa)P - \kappa \beta_{e,P} e_i}{\alpha v_U} - e_i \quad (4.5)$$

and

$$x_I^*(\theta, P, e_i) = \frac{\theta - P}{\alpha v_\epsilon} - e_i, \quad (4.6)$$

respectively, where $\kappa = \sigma_{\theta,P}/[v_P - \frac{\sigma_{e,P}^2}{v_e}]$ and $v_U = v_D\left(1 - \frac{\rho_{P,D}^2}{1-\rho_{e,P}^2}\right) = v_D - \kappa\sigma_{\theta,P}$, also $\beta_{e,P} = \sigma_{e,P}/v_e$, $\sigma_{e,P} = \mathbb{C}ov[\tilde{e}_i, \tilde{P}]$ and $\sigma_{\theta,P} = \mathbb{C}ov[\tilde{\theta}, \tilde{P}]$.

We now compute expected utilities for the informed and uninformed traders, i.e., $V_I(\lambda, e_i)$ and $V_U(\lambda, e_i)$. First, trader i's welfare conditional on his information set is given by

$$\mathbb{E}\left[-e^{-\alpha \tilde{W}_i}|\mathcal{F}_i\right] = -\exp\left\{-\alpha e_i P - \frac{1}{2}\frac{\chi_i^2}{v_i}\right\}, \quad (4.7)$$

where $\chi_i \equiv \mathbb{E}\left[\tilde{R}|\mathcal{F}_i\right]$ and $v_i \equiv \mathbb{V}ar\left[\tilde{R}|\mathcal{F}_i\right]$ are the expectation and variance of return conditional on trader i's information set. Since, conditional on the endowment shock e_i, the price P and conditional expected return χ_i follow a bivariate normal distribution, we can obtain the following expression for trader i's welfare given his own endowment shock.

Proposition 4.1.

$$V_K(\lambda, e_i) = -V_K(\lambda)\exp\left\{\frac{1}{2}A(\lambda)e_i^2\right\}, \qquad V_K(\lambda) = \frac{1}{\sqrt{1+\xi_K(\lambda)}}, \tag{4.8}$$

where

$$A(\lambda) = \frac{\alpha^2(v_{P|e}v_D - \sigma^2_{\theta,P}) - \beta^2_{e,P} - 2\alpha(v_D - \sigma_{\theta,P})\beta_{e,P}}{v_{P|e} + v_D - 2\sigma_{\theta,P}},$$

and

$$\xi_K(\lambda) = \begin{cases} \left(v_{P|e} + v_\theta - 2\sigma_{\theta,P}\right)/v_e, & \mathcal{F}_i = \{\theta, P, e_i\}; \\ \left(v_{P|e} + (v_D - v_U) - 2\sigma_{\theta,P}\right)/v_U, & \mathcal{F}_i = \{P, e_i\}, \end{cases} \qquad v_{P|e} = v_P - \frac{\sigma^2_{e,P}}{v_z + v_u}.$$

From Proposition 4.1, the relative gain in expected utility of being informed is *independent* of the realized endowment shock e_i, since

$$\gamma(\lambda) = \frac{V_I(\lambda, e_i) - V_U(\lambda, e_i)}{-V_U(\lambda, e_i)} = 1 - \sqrt{\frac{v_\epsilon}{v_D - \kappa\sigma_{\theta,P}}}. \tag{4.9}$$

Interestingly, the solution to trader i's optimization problem as in (4.2) does not depend on the trader-specific endowment shock and boils down to (2.9), just as in the baseline model. Also, the concavity condition in p_i, $\mathcal{U}''(p_i; \lambda, e_i) < 0$, is satisfied under Assumption 2.2.

4.2. **Equilibrium.** Since the risky asset is in zero net supply, market clearing requires

$$\int_0^1 \left[\lambda x_I^*(\theta, P, e_i) + (1-\lambda)x_U^*(P, e_i)\right] di = 0, \tag{4.10}$$

where λ is the fraction of informed traders. Next proposition determines the coefficient b_θ and b_z, and the level of informed trading λ in equilibrium.

Proposition 4.2. *Under Assumption 2.2, for given $\lambda \in [0,1]$, let $\Psi \equiv v_z/(v_z + v_u)$. Then there exists a linear equilibrium price of the risky asset,*

$$\tilde{P} = b_\theta\tilde{\theta} - b_z\tilde{z}, \tag{4.11}$$

where

$$b_\theta = \frac{1}{1 + x^{-2}\frac{v_\theta^{-1} + \lambda v_\epsilon^{-1}}{v_u^{-1} + v_z^{-1}}} + \frac{\lambda}{\frac{v_\epsilon + \lambda v_\theta}{v_\epsilon + v_\theta} + x^2\frac{v_u^{-1} + v_z^{-1}}{v_\epsilon^{-1} + v_\theta^{-1}}}, \tag{4.12}$$

$x \equiv b_\theta/b_z$ solves

$$x = \frac{1}{\alpha v_\epsilon}\left(\lambda + \frac{1-\lambda}{\Psi^{-1} + x^{-2}(v_\epsilon^{-1} + v_\theta^{-1})v_u}\right), \tag{4.13}$$

and λ is the solution of

$$\alpha\mu\, c'(\lambda) = \frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)}, \tag{4.14}$$

where $\gamma(\lambda)$ is given by (4.9)*.*

Proposition 4.2 shows that the equilibrium information acquisition is exactly the same as in the baseline model, while the equilibrium price shares the same linear structure but with more complicated coefficients depending on the size of the endowment shock, in addition to other parameters in the baseline model.

4.3. **Welfare.** The welfare of trader i, assuming $c(p) = p^2$, given his endowment shock e_i, can be measured by

$$\mathcal{W}(\lambda; e_i) \equiv \mathcal{U}(p^*; \lambda, e_i) = \bar{V}(p^*, \lambda)e^{\Phi(p^*,\lambda,e_i)}, \qquad p^* \equiv p^*(\lambda) = \lambda, \tag{4.15}$$

where

$$\bar{V}(p^*, \lambda) = p^* V_I(\lambda) + (1-p^*)V_U(\lambda)$$

and

$$\Phi(p^*, \lambda, e_i) = \frac{p^*}{2}\frac{\gamma(\lambda)}{1-p^*\gamma(\lambda)} + \frac{1}{2}A(\lambda)e_i^2.$$

Comparing to the baseline setting, the welfare function shows an extra welfare cost, $A(\lambda)e_i^2/2$, which characterizes the cost of the hedging on the endowment risk. It increases with the size of the endowment shock. Intuitively, a trader with a relatively large endowment shock (e_i) trades to hedge endowment risk more than to speculate on the future payoff. Thus, the hedging component can dominate the speculative component when the endowment shock is large (as in (4.3)). As a result, in equilibrium, traders with relatively large endowment shocks (hedgers) must compensate those with smaller endowment shocks (speculators) for supplying liquidity to absorb the hedging demand.

As in the baseline model, we decompose the marginal welfare into information-gain effect and informed trading effect as follows,

$$\frac{\mathcal{W}'(\lambda; e_i)}{-\mathcal{W}(\lambda; e_i)} = \underbrace{\frac{1}{-\mathcal{U}}\frac{\partial \mathcal{U}}{\partial p^*}}_{\text{information-gain effect}} + \underbrace{\frac{1}{-\mathcal{U}}\frac{\partial \mathcal{U}}{\partial \lambda}}_{\text{informed-trading effect}}, \tag{4.16}$$

where the information-gain effect

$$\frac{1}{-\mathcal{U}}\frac{\partial \mathcal{U}}{\partial p^*} = \frac{\gamma(\lambda)(1-2\lambda\gamma(\lambda))}{2(1-\lambda\gamma(\lambda))^2} \tag{4.17}$$

and the informed-trading effect

$$\frac{1}{-\mathcal{U}}\frac{\partial \mathcal{U}}{\partial \lambda} = \underbrace{-\frac{1}{2}A'(\lambda)e_i^2}_{\text{Hirshleifer effect}} - \left[\Gamma(\lambda)\frac{V_I'}{V_I} + (1-\Gamma(\lambda))\frac{V_U'}{V_U}\right], \tag{4.18}$$

where $\Gamma(\lambda) = \frac{\lambda(1-2\lambda\gamma)(1-\gamma)}{2(1-\lambda\gamma)^2}$. The marginal welfare decomposition shows that, while the information-gain effect has the identical expression as in the baseline model, the informed-trading effect has an extra component reflecting the *Hirshleifer effect*, which measures the increase in cost of hedging endowment risk due to increase in informed trading (λ). The Hirshleifer effect is a further impediment to risk-sharing. Intuitively, informed trading reduces the incentives to trade of the speculators (traders with little or no endowment shocks), thus it becomes more expensive for hedgers (traders with larger endowment shocks) to execute their market orders. In other words, informed trading increases the *cost of liquidity.* Due to the complex nature of the equilibrium price, it is generally very difficult to analytically derive market conditions for welfare improvement. Next, we consider two special cases where this is possible.

First, we compare welfare between the no-information equilibrium ($\lambda = 0$) and the full-information equilibrium ($\lambda = 1$). Note that for $\lambda = 0$, the equilibrium price becomes $\tilde{P} = \alpha v_D \tilde{z}$ and traders' optimal portfolios are given by $x_U^*(P, e_i) = \frac{-P}{\alpha v_D} - e_i$. On the other hand, when $\lambda = 1$, the equilibrium price and trader i's optimal portfolio are given by $\tilde{P} = \tilde{\theta} - \alpha v_\epsilon \tilde{z}$ and $x_I^*(\theta, P, e_i) = \frac{\theta - P}{\alpha v_\epsilon} - e_i$, respectively. The following proposition compares traders' welfare in the two equilibria.

Proposition 4.3. *Given the welfare of trader i characterized by* (4.8)*, trader i is always better off at the no-informed-trading than at the full-informed-trading equilibrium, i.e.,* $\mathcal{W}(0; e_i) > \mathcal{W}(1; e_i)$ *for any* e_i*.*

The result in Proposition 4.3 is consistent with the noise-trader model. Note that the difference in welfare is purely due to the reduction in payoff uncertainty since traders have symmetric information in both equilibria. As explained by Goldstein and Yang (2017), "*disclosure harms investors through destroying trading opportunities*".

For the second special case, we take the limit as $v_u \to \infty$, thus endowment shocks are no longer informative about the private signal $\tilde{\theta}$, and equilibrium converges to that in the noise-trader model. In this case, we can extend the result in Corollary 3.1 at the no-information equilibrium to reflect the Hirshleifer effect.

Proposition 4.4. *Assume* $v_u \to \infty$*, information-acquisition improves trader i's welfare at the no-information-acquisition equilibrium (i.e.,* $\mathcal{W}'(0; e_i) \geq 0$*) if and only if*

$$\underbrace{-\frac{n\xi_0}{1+\xi_0}\left[1+\frac{e_i^2}{1+\xi_0}\right]}_{\textit{informed-trading effect}} + \underbrace{\frac{1}{2}\left(1-\frac{1}{\sqrt{1+n}}\right)}_{\textit{information-gain effect}} > 0. \tag{4.19}$$

Proposition 4.4 shows that the Hirshleifer effect has a *negative* effect on marginal welfare, and the welfare loss increases with information quality and risk-sharing incentive. This shows that informed trading is more detrimental to the welfare of those traders with relatively large endowment shocks. Therefore, in order for marginal welfare to be positive, the anticipatory welfare benefit of information-gain must be large enough to offset the cost of informed trading due to both return and Hirshleifer effects. Next, we numerically examine whether the result is robust for $v_u \in (0, \infty)$.

4.4. **Numerical Analysis.** In Figure 4.1, Panels (A) and (B) show the parameter regions of (n, ξ_0) in which information acquisition is welfare improving at the no-information equilibrium with $\lambda = 0$ for different values of v_u and e. We observe that marginal welfare $\mathcal{W}'(0, e)$ is more likely to be positive when endowment shock,

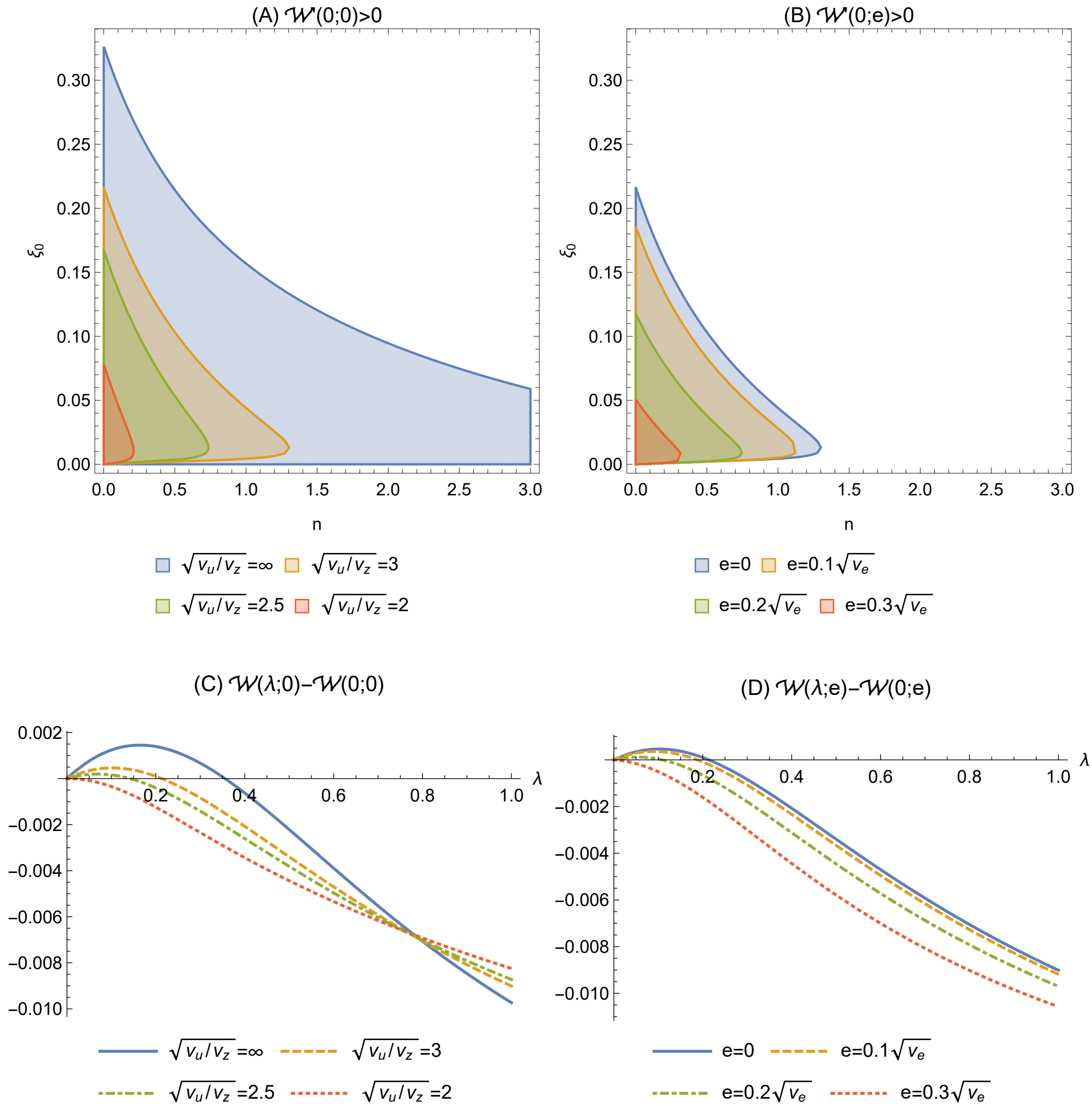

FIGURE 4.1. Panels A and B show the parameter regions of (n, ξ_0) in which $\mathcal{W}'(0;e) > 0$, where $e = 0$ and $\sqrt{v_u/v_z} \in \{\infty, 3, 2.5, 2\}$ in Panel A, and $e/\sqrt{v_e} \in \{0, 0.1, 0.2, 0.3\}$ and $\sqrt{v_u/v_z} = 3$ in Panel B. Panels C and D show the welfare improvement $\mathcal{W}(\lambda;e) - \mathcal{W}(0;e)$, where $n = 0.1$ and $\xi_0 = 0.05$.

e, and the idiosyncratic variance of endowment shock, v_u, are small, which has the following implications.

First, information acquisition is more likely to hurt those traders who demand liquidity, whose optimal demand x_i^* is mostly driven by the endowment shock e_i rather than the speculative component. Intuitively, informed trading distorts risk-sharing and increases the hedging cost for endowment risk, which has a negative effect on their welfare. Therefore, for those traders with large hedging demands, the Hirshleifer effect can dominate the information-gain effect.

Second, information acquisition is more likely to hurt traders' welfare when individual trader's endowment shock e_i is more informative about the aggregate endowment shock $\tilde{z}$. Intuitively, taking λ as given, a higher correlation between e_i and $\tilde{z}$ make price more informative. Therefore, when more payoff uncertainty is resolved, it brings more distortion to risk-sharing, which enhances both the Hirshleifer and return effects, resulting a large welfare cost.

Finally, Panels (C) and (D) plot the welfare function for different values of v_u and e. We find that the optimal state λ^*, where welfare $\mathcal{W}(\lambda^*, e_i)$ is maximized, is decreasing in e_i. Moreover, for traders with a sufficiently large e_i, $\lambda^* = 0$ (no-information equilibrium) becomes the optimal state. Therefore, information acquisition can have different welfare effects on traders, depending on the size of their endowment shocks. In particular, information acquisition can be welfare-improving for speculators who provide liquidity but welfare-reducing for hedgers who demand liquidity.

4.5. **Policy Implications.** In terms of policy implications, the numerical results show that the Pareto-optimal state is *not* unique in the endowment economy, where noise trading is endogenized. This is because the relationship between welfare $\mathcal{W}(\lambda; e_i)$ and the state variable λ is different for traders with different endowment shock e_i. Consider the state λ_0^* where $\mathcal{W}'(\lambda_0^*, 0) = 0$ (so that the welfare is maximized) for pure speculators who have no endowment risk, are thus not affected by Hirshleifer effect. In this case, λ_0^* is the highest level of informed trading, above which an increase in informed-trading will be harmful to the welfare of *every* trader, i.e., $\mathcal{W}'(\lambda, e_i) < 0$ for $\lambda > \lambda_0^*$ and $i \in (0, 1)$. Hence, this means that *any* $\lambda^* \in [0, \lambda_0^*]$ is a Pareto-optimal state, since deviating from λ^* will be welfare improving for some traders and welfare reducing for others. For example, in Panel (D), the optimal state for a pure speculator is approximately $\lambda_0^* = 0.2$. Assume the current state $\lambda < \lambda_0^*$, an increase in λ would make the pure speculator better off, i.e., $\mathcal{W}'(\lambda; 0) > 0$, however, it would be welfare reducing for the trader (hedger) with $e_i = 0.3\sqrt{v_e}$, i.e.,

$\mathcal{W}'(\lambda; 0.3\sqrt{v_e}) < 0$. On the other hand, if the current state $\lambda > \lambda_0^*$, a marginal reduction in λ is Pareto-improving since, in this case, $\mathcal{W}'(\lambda; e_i) < 0$ for all traders.

In the noise-trader model, making information acquisition less costly to encourage information acquisition does not necessarily improve traders' welfare. The above analysis indicates that, with endowment shocks, decreasing cost sensitivity never leads to Pareto-improvement. However, information acquisition can still be valuable for traders with small endowment shock, i.e., the liquidity providers.

In general, our results suggest that, in an economy with endowment shocks, regulators should never encourage information acquisition which leads to more informed trading, since doing so will hurt traders with relatively large endowment risks and high liquidity demand. Moreover, regulators should only discourage information acquisition when there is excessive informed trading in the market, i.e., $\lambda > \lambda_0^*$, because otherwise doing so will be welfare-reducing for speculators with relatively small endowment risks who provide liquidity.

5. Conclusion

In a standard pure-exchange economy with rational expectations when information choices are made probabilistically, we find that information acquisition has two opposing effects on traders' marginal welfare, namely, the positive information-gain effect and the negative informed-trading effect. The former is welfare-improving whereas the latter is welfare-reducing, the net effect on marginal welfare depends on the information quality, risk-sharing incentives and the cost sensitivity of information acquisition.

In a noise-trader model, utility-maximizing speculators provide liquidity to noise traders whose random demand is exogenously given. We find that the information-gain effect tends to dominate the informed-trading effect in marginal welfare when both risk-sharing incentives and information quality are low. In a more generalized model with endowment shocks, traders' demand consists of both speculative and hedging components. In this case, the welfare trade-off between information-gain and informed-trading effects depends on the size of the endowment shock, and traders with large endowment shocks tend to experience a welfare reduction whereas those with small endowment shocks experience a welfare improvement.

Overall, our results suggest that information acquisition can be very valuable, especially to liquidity providers when risk-sharing incentives are low. Therefore, regulations to level the playing field by minimizing information-asymmetry between traders might actually be welfare-reducing for a significant proportion of traders.

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Appendix A. Proofs

To simplify the notation, we drop the trader-specific subscript i for all proofs in this appendix.

A.1. **Proof of Proposition 2.1.** (i). The CARA-normal setting implies that traders' optimal portfolio choices at time $t = 1$ follow the standard mean-variance type,

$$x_K^* = \begin{cases} x_I^*(\theta, P) = \dfrac{\mathbb{E}[\tilde{R}|\theta, P]}{\alpha \mathbb{V}ar[\tilde{R}|\theta, P]}, & \mathcal{F}_I = \{\theta, P\}; \\ x_U^*(P) = \dfrac{\mathbb{E}[\tilde{R}|P]}{\alpha \mathbb{V}ar[\tilde{R}|P]}, & \mathcal{F}_U = \{P\}. \end{cases} \tag{A.1}$$

The linear equilibrium price for the risky asset, $\tilde{P}$, has the following structure

$$\tilde{P} = b_\theta \tilde{\theta} - b_z \tilde{z}, \tag{A.2}$$

where the coefficients b_θ and b_z are determined in equilibrium. Thus, we have

$$\tilde{P} \sim \mathcal{N}(0, v_P), \qquad v_P = b_\theta^2 v_\theta + b_z^2 v_z$$

and

$$\tilde{D}|P \sim \mathcal{N}(\beta_{P,\theta} P, v_{\theta|P}), \qquad \beta_{P,\theta} = \frac{\sigma_{\theta,P}}{v_P}, \qquad v_{\theta|P} = v_\epsilon + (1 - \rho_{\theta,P}^2) v_\theta.$$

We introduce some notations; for any two random variables $\tilde{x}$ and $\tilde{y}$,

$$\begin{aligned} & v_x \equiv \mathbb{V}ar[\tilde{x}], \;\; v_y \equiv \mathbb{V}ar[\tilde{y}], \;\; \sigma_{x,y} \equiv \mathbb{C}ov[\tilde{x}, \tilde{y}], \\ & \beta_{x,y} \equiv \frac{\sigma_{x,y}}{v_x}, \;\; \rho_{x,y} \equiv \frac{\sigma_{x,y}}{\sqrt{v_x v_y}}, \;\; v_{x|y} = \mathbb{V}ar[\tilde{x}|y]. \end{aligned} \tag{A.3}$$

Thanks to these new notations, it is easy to see that (A.1) can be rewritten as follows

$$x_K^* = \begin{cases} x_I^*(\theta, P) = \frac{\theta - P}{\alpha v_\epsilon}, & \mathcal{F}_K = \{\theta, P\}; \\ x_U^*(P) = \frac{-P}{\alpha v_U}, & \mathcal{F}_K = \{P\}, \end{cases} \tag{A.4}$$

where

$$v_U = \frac{v_{\theta|P}}{1 - \beta_{P,\theta}} = \frac{v_\epsilon + \frac{b_z^2 v_z}{b_\theta^2 v_\theta + b_z^2 v_z} v_\theta}{1 - \frac{b_\theta v_\theta}{b_\theta^2 v_\theta + b_z^2 v_z}}.$$

Next, to solve for the equilibrium price P, we apply the market clearing condition, $\lambda x_I^*(\theta, P) + (1 - \lambda) x_U^*(P) = \tilde{z}$, and obtain the equilibrium price, $\tilde{P} = \frac{\lambda \bar{v}}{v_\epsilon} \tilde{\theta} - \alpha \bar{v} \tilde{z}$, where $1/\bar{v} = \lambda / v_\epsilon + (1 - \lambda)/v_U$. By matching coefficient to the conjectured equilibrium price, we obtain $b_\theta = \frac{\lambda \bar{v}}{v_\epsilon}$ and $b_z = \alpha \bar{v}$. Since $b_\theta = \lambda b_z / (\alpha v_\epsilon)$, we obtain an explicit solution for $\bar{v}$ by solving

$$\frac{\lambda}{v_\epsilon} + \frac{(1-\lambda)\left(1 - \frac{(\lambda b_z/\alpha)\, v_\theta/v_\epsilon}{(\lambda b_z/\alpha)^2 v_\theta/v_\epsilon^2 + b_z^2 v_z}\right)}{v_\epsilon + \frac{b_z^2 v_z}{(\lambda b_z/\alpha)^2 v_\theta/v_\epsilon^2 + b_z^2 v_z} v_\theta} = \frac{b_z}{\alpha}$$

for b_z and substituting the solution back into the expression for v_U and $\bar{v}$.

(ii). By substituting $b_\theta = (\lambda\bar{v}/v_\epsilon)$ and $b_z = \alpha\bar{v}$ back into traders' optimal demand functions in (A.4) we obtain $x_I^*(\theta, P)$ and $x_U^*(P)$ in (2.4).

A.2. **Proof of Lemma 2.2.** Given the explicit solution for b_θ and b_z, the expected utilities of the informed and uninformed traders are given by

$$V_I(\lambda) = V_U(\lambda)(1-\gamma(\lambda)), \qquad V_U(\lambda) = -\frac{1}{\sqrt{1+\xi(\lambda)}},$$

where

$$(1-\gamma(\lambda))^2 = \left(\frac{V_I(\lambda)}{V_U(\lambda)}\right)^2 = \frac{\lambda^2 n(n+1) + \xi_0}{(n+1)\,(\lambda^2 n + \xi_0)} \tag{A.5}$$

and

$$\xi(\lambda) = \frac{\mathbb{V}ar[\mathbb{E}(\tilde{R}|P)]}{\mathbb{V}ar[\tilde{R}|P]} = \frac{\xi_0^2\,(\lambda^2 n + \xi_0)}{(\lambda^2 n(n+1) + \xi_0(\lambda n+1))^2}, \qquad \xi(0) = \xi_0. \tag{A.6}$$

Note that $0 < \gamma(\lambda) < 1$,

$$\frac{V_I'}{V_I} = \frac{V_U'}{V_U} - \frac{\gamma'(\lambda)}{1-\gamma(\lambda)}, \qquad V_U' = \frac{1}{2(\sqrt{1+\xi(\lambda)})^3}\xi'(\lambda),$$

$$-(1-\gamma(\lambda))\gamma'(\lambda) = \frac{2\lambda n^2(1+n)\xi_0}{[(1+n)(n\lambda^2+\xi_0)]^2} > 0$$

and

$$\xi'(\lambda) = -2n\xi_0^2 \frac{\xi_0^2 + \lambda\xi_0(1+2n) + n(1+n)\lambda^3}{(\lambda^2 n(n+1) + \xi_0(\lambda n+1))^3} < 0,$$

hence $\gamma'(\lambda) < 0$ and $V_U' < 0, V_I' < 0$.

A.3. **Proof of Lemma 2.3.** Let $\bar{V}(\lambda) \equiv \lambda V_I(\lambda) + (1-\lambda)V_U(\lambda)$ and $\bar{V}(p;\lambda) \equiv pV_I(\lambda) + (1-p)V_U(\lambda)$, then

$$\mathcal{U}'(p;\lambda) = e^{\alpha\mu c(p)}[\alpha\mu c'(p)\bar{V}(p,\lambda) + [V_I(\lambda) - V_U(\lambda)]]$$

$$\mathcal{U}''(p;\lambda) = \alpha\mu c'(p) e^{\alpha\mu c(p)}\left[\left(\alpha\mu c'(p) + \frac{c''(p)}{c'(p)}\right)\bar{V}(p,\lambda) + 2[V_I(\lambda) - V_U(\lambda)]\right].$$

Therefore, the necessary and sufficient condition for $\mathcal{U}''(p;\lambda) \le 0$ is

$$\left(\alpha\mu c'(p) + \frac{c''(p)}{c'(p)}\right)\bar{V}(p,\lambda) + 2[V_I(\lambda) - V_U(\lambda)] \le 0,$$

or, put differently,

$$\left(\alpha\mu c'(p) + \frac{c''(p)}{c'(p)}\right) V_U[1 - p\gamma] - 2V_U\gamma \leq 0.$$

Call $K(p) = (\alpha\mu c'(p) + c''(p)/c'(p))$, then such condition is equivalent to

$$K(p)[1 - p\gamma] - 2\gamma \geq 0 \iff \gamma(\lambda) \leq \frac{K(p)}{2 + K(p)p}, \tag{A.7}$$

where the latter inequality must hold true for any p. This proves (2.11). Now, if $c(p) = p^2$, the r.h.s. in (A.7) is decreasing in p. Therefore, under this assumption, (A.7) is equivalent to

$$\gamma(\lambda) \leq \frac{K(1)}{2 + K(1)} = \frac{2\alpha\mu + 1}{2\alpha\mu + 3}.$$

A.4. **Proof of Proposition 2.4.** Note that if the concavity condition, $\mathcal{U}''(p;\lambda) \leq 0$, is satisfied, the Nash equilibrium for the choice of probability p to observe the private signal $\tilde{\theta}$ must be symmetric, since traders are homogeneous, i.e., $p^* = \lambda$ for all traders, from which we obtain

$$\alpha\mu = -\frac{1}{c'(\lambda)} \frac{V_I(\lambda) - V_U(\lambda)}{\bar{V}(\lambda)} = \frac{1}{c'(\lambda)} \frac{\gamma(\lambda)}{1 - \lambda\gamma(\lambda)}.$$

Substitute this expression into

$$\gamma(\lambda) \leq \frac{2\alpha\mu + 1}{2\alpha\mu + 3},$$

then it can be written as

$$\gamma(\lambda)^2(3\lambda^2 - 1) + \gamma(\lambda)(1 - 3\lambda - \lambda^2) + \lambda \geq 0.$$

A tedious algebraic derivation shows that this is satisfied as soon as $\gamma(\lambda) \leq 1/2$. Moreover, since $\gamma(\lambda)$ is a decreasing function, it is sufficient to have $\gamma(0) \leq 1/2$. Finally, recall that $\gamma(0)$ can be written as a function on n. Specifically, we obtain

$$\gamma(0) = 1 - \frac{1}{\sqrt{1+n}} \leq \frac{1}{2} \iff n \leq 3.$$

In other words, for $c(p) = p^2$, $n \leq 3$ guarantees $\gamma(\lambda) \leq 1/2$, which is a sufficient condition for $\mathcal{U}''(p;\lambda) \leq 0$ given any equilibrium $\lambda \in [0,1]$, hence $p^* = \lambda$, and λ in equilibrium is given by (2.13).

The proof on the existence and uniqueness of the Nash equilibrium p^* with respect to parameter μ is provided in Appendix B.

A.5. **Proof of Proposition 3.1.** On the information-gain effect, it follows directly from $\frac{\partial\Phi(p^*,\lambda)}{\partial p^*} = \frac{\gamma(\lambda)}{2(1-p^*\gamma(\lambda))^2}$ and (3.2) that

$$\frac{1}{-\mathcal{U}}\frac{\partial \mathcal{U}}{\partial p^*} = \frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)} - \frac{\gamma(\lambda)}{2(1-p^*\gamma(\lambda))^2},$$

leading to the information-gain effect.

On the informed-trading effect, it follows from $\frac{\partial\Phi(p^*,\lambda)}{\partial\lambda} = \frac{p^*\gamma'(\lambda)}{2(1-p^*\gamma(\lambda))^2}$ and (3.2) that, in equilibrium,

$$\frac{1}{-\mathcal{U}}\frac{\partial \mathcal{U}}{\partial \lambda} = -\frac{V_U'}{V_U} + \frac{\lambda\gamma'(\lambda)}{1-\lambda\gamma(\lambda)} - \frac{\lambda\gamma'(\lambda)}{2(1-\lambda\gamma(\lambda))^2}.$$

This, together with

$$\gamma'(\lambda) = -\frac{1}{1-\gamma(\lambda)}\left[\frac{V_I'}{V_I} - \frac{V_U'}{V_U}\right],$$

leads to the informed-trading effect with $\Gamma(\lambda) = \frac{\lambda(1-2\lambda\gamma)(1-\gamma)}{2(1-\lambda\gamma)^2}$.

A.6. **Proof of Corollary 3.1.** (i). Applying condition (3.3) to $\lambda = 0$, we have

$$\frac{V_U'(0)}{V_U(0)} < \frac{1}{2}\gamma(0). \tag{A.8}$$

Note that

$$\frac{V_U'(0)}{V_U(0)} = -\frac{\xi'(0)}{2(1+\xi_0)}, \qquad \xi'(0) = -2n\xi_0, \qquad \gamma(0) = 1 - \frac{1}{\sqrt{1+n}}.$$

This, together with (A.8), leads to (3.4).

(ii). Condition (3.4) can be written as

$$\frac{\xi_0}{1+\xi_0} < \frac{1}{2n}\left(1 - \frac{1}{\sqrt{1+n}}\right) := \xi_0^*(n).$$

It can be verified that $\xi_0^*(n)$ is a decreasing function of n. Note that $\lim_{n\to 0}\xi_0^*(n) = \frac{1}{4}$, therefore $\xi_0 \le 1/3$ becomes a necessary condition for $\mathcal{W}'(0) \ge 0$.

Lemma A.1. *Let $X \in \mathbb{R}^n$ be a normally distributed random vector with mean μ and variance-covariance matrix Σ. Let $b \in \mathbb{R}^n$ be a given vector, and $A \in \mathbb{R}^{n\times n}$ a symmetric matrix. If $I - 2\Sigma A$ is positive definite, then $\mathbb{E}\left[\exp\{b^\top X + X^\top AX\}\right]$ is well defined, and given by*

$$\mathbb{E}\left[\exp\{b^\top X + X^\top AX\}\right] = |I - 2\Sigma A|^{-1/2}\exp\{b^\top\mu + \mu^\top\Sigma\mu + \frac{1}{2}(b+2A\mu)^\top(I-2\Sigma A)^{-1}\Sigma(b+2A\mu)\}.$$

A.7. **Proof of Proposition 4.1.** First, note that each trader's expected utility conditional on his information set is given by

$$V(\lambda;e) \equiv \mathbb{E}\left\{\mathbb{E}\left[-e^{-\alpha\tilde{W}}|\mathcal{F}\right]|e\right\} = \mathbb{E}\left[-\exp\left\{-\alpha eP - \frac{1}{2}\frac{\chi^2}{v}\right\}|e\right], \qquad K \in \{I,U\}, \quad \text{(A.9)}$$

where $\chi \equiv \mathbb{E}[\tilde{R}|\mathcal{F}]$ and $v \equiv \mathbb{V}ar[\tilde{R}|\mathcal{F}]$, respectively. First, since (given the endowment shock e) χ and P follow a bivariate normal distribution with mean vector and covariance matrix given by

$$\mu = \begin{pmatrix} \mu_{\chi|e} \\ \mu_{P|e} \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} v_{\chi|e} & \sigma_{(\chi,P)|e} \\ \sigma_{(\chi,P)|e} & v_{P|e} \end{pmatrix}, \quad \text{(A.10)}$$

where $\mu_{\chi|e} \equiv \mathbb{E}[\chi|e]$, $\mu_{P|e} \equiv \mathbb{E}[P|e]$, $v_{\chi|e} \equiv \mathbb{V}ar[\chi|e]$, $v_{P|e} \equiv \mathbb{V}ar[P|e]$ and $\sigma_{(\chi,P)|e} \equiv \mathbb{C}ov[\chi, P|e]$. Thus, using Lemma A.1 we can establish the following result,

$$V(\lambda;e) = -\exp\left\{-\frac{\mu^2_{\chi|e} + \alpha e\left[2\nu\mu_{P|e} + 2\mu_{\chi|e}\sigma_{(\chi,P)|e} + \alpha e\left(\sigma^2_{(\chi,P)|e} - \nu v_{P|e}\right)\right]}{2\nu}\right\}\sqrt{\frac{1}{1+\xi_K}}, \quad \text{(A.11)}$$

where $\nu = v + v_{\chi|e}$. If the trader is informed, i.e., $\mathcal{F} = \{\theta, P, e\}$, since $\chi = \theta - P$ and $v = v_\epsilon$, we obtain that

$$\mu_{\chi|e} = -\beta_{e,P}e, \;\; \mu_{P|e} = \beta_{e,P}e, \;\; v_{\chi|e} = v_\theta + v_{P|e} - 2\sigma_{\theta,P}, \;\; \sigma_{(\chi,P)|e} = \sigma_{\theta,P} - v_{P|e}. \quad \text{(A.12)}$$

Substituting (A.12) into (A.11) leads to the expected utility of an informed trader in (4.8) with $v = v_\epsilon$. On the other hand, if the trader is uninformed, i.e., $\mathcal{F} = \{P, e\}$, since $\chi = (1-\kappa)(-P) - \kappa\beta_{e,P}e$ and $v = v_D - \kappa\sigma_{\theta,P}$, $\kappa = \sigma_{\theta,P}/v_{P|e}$, we obtain that

$$\mu_{\chi|e} = -\beta_{e,P}e, \;\; \mu_{P|e} = \beta_{e,P}e, \;\; v_{\chi|e} = (1-\kappa)^2 v_{P|e}, \;\; \sigma_{(\chi,P)|e} = -(1-\kappa)v_{P|e}. \quad \text{(A.13)}$$

Substituting (A.13) into (A.11) leads to the expected utility of an uninformed trader in (4.8) with $v = v_D - \kappa\sigma_{\theta,P}$.

A.8. **Proof of Proposition 4.2.** Substituting the optimal demands $x^*_U(P,e)$ and $x^*_I(\theta,P,e)$ in (4.5) and (4.6) into the market clearing condition (4.10) leads to the following,

$$\frac{(-P)}{\alpha\bar{v}} + \frac{\lambda}{\alpha v_\epsilon}\tilde{\theta} = \left[1 + (1-\lambda)\frac{\kappa\beta_{e,P}}{\alpha v_U}\right]\tilde{z}, \quad \text{(A.14)}$$

where $\frac{1}{\bar{v}} \equiv \frac{\lambda}{v_\epsilon} + \frac{1-\lambda}{v_U/(1-\kappa)}$. Thus, the equilibrium price can be written as

$$P = \underbrace{\frac{\lambda\bar{v}}{v_\epsilon}}_{b_\theta}\tilde{\theta} - \underbrace{\alpha\bar{v}\left[1 + (1-\lambda)\frac{\kappa\beta_{e,P}}{\alpha v_U}\right]}_{b_z}\tilde{z}. \tag{A.15}$$

Therefore, we obtain

$$x \equiv \frac{b_\theta}{b_z} = \frac{1}{\alpha v_\epsilon}\frac{\lambda}{1 + (1-\lambda)\frac{\kappa\beta_{e,P}}{\alpha v_U}},$$

which can be written as

$$x = \frac{1}{\alpha v_\epsilon}\left[\lambda - (1-\lambda)\left(\kappa\beta_{e,P}\frac{v_\epsilon}{v_U}\right)x\right]. \tag{A.16}$$

Since $v_U = v_D - \kappa\sigma_{\theta,P}$, $\kappa = \sigma_{\theta,P}/v_{P|e}$ and $\beta_{e,P} = \sigma_{e,P}/v_e$, also,

$$\sigma_{e,P} = -b_z v_z,\ \ \sigma_{\theta,P} = b_\theta v_\theta,\ \ v_{P|e} = b_\theta^2 v_\theta + b_z^2 v_{z|e},\ \ v_{z|e} = (v_z^{-1} + v_u^{-1})^{-1}, \tag{A.17}$$

we can obtain that

$$-\left(\kappa\beta_{e,P}\frac{v_\epsilon}{v_U}\right)x = \frac{v_z v_\epsilon v_\theta x^2}{v_u v_z v_D + v_e v_\epsilon v_\theta x^2} = \frac{1}{v_e/v_z + x^{-2}v_u(v_\theta^{-1} + v_\epsilon^{-1})}. \tag{A.18}$$

Substituting (A.18) back into (A.16) leads to (4.13). Next, given x, we substitute (A.17) into the expression for b_θ and obtain that

$$b_\theta = \frac{\lambda\bar{v}}{v_\epsilon} = \frac{\lambda b_z\left(v_u v_z v_D + v_e v_\epsilon v_\theta x^2\right)}{b_z x^2 v_z v_\epsilon v_\theta - v_e v_\epsilon v_\theta x(1-\lambda) + b_z v_u(v_\epsilon v_\theta x^2 + v_z v_D\lambda)}. \tag{A.19}$$

Since $b_z = b_\theta/x$, (A.19) can be simplified to $b_\theta = \frac{v_e v_\epsilon v_\theta x^2 + v_u v_z v_D\lambda}{v_e v_\epsilon v_\theta x^2 + v_u v_z(v_\epsilon + v_\theta\lambda)}$, which leads to the expression in (4.12).

A.9. **Proof of Corollary 4.3.** For $\lambda = 0$, since the equilibrium price $\tilde{P} = \alpha v_D\tilde{z}$, we have

$$v_{P|e} = \alpha^2 v_D^2 v_{z|e},\ \ \sigma_{\theta,P} = 0,\ \ \beta_{e,P} = -\alpha v_D\frac{v_z}{v_e},\ \ \nu = v_D(1 + \alpha^2 v_D v_{z|e}). \tag{A.20}$$

Substituting (A.20) into (4.8) leads to

$$A = \frac{\alpha^2(v_\epsilon + v_\theta)\left(2v_z/v_e - (v_z/v_e)^2 + \alpha^2(v_\epsilon + v_\theta)v_{z|e}\right)}{1 + \alpha^2 v_{z|e}(v_\epsilon + v_\theta)}, \qquad \frac{1}{1+\xi_U} = 1 + \alpha^2(v_\epsilon + v_\theta)v_{z|e} \tag{A.21}$$

On the other hand, for $\lambda = 1$, since the equilibrium price $\tilde{P} = \tilde{\theta} - \alpha v_\epsilon\tilde{z}$, we have

$$v_{P|e} = v_\theta + \alpha^2 v_\epsilon^2 v_{z|e},\ \ \sigma_{\theta,P} = v_\theta,\ \ \beta_{e,P} = -\alpha v_\epsilon\frac{v_z}{v_e},\ \ \nu = v_\epsilon(1 + \alpha^2 v_\epsilon v_{z|e}). \tag{A.22}$$

Substituting (A.22) into (4.8) leads to

$$A = \frac{\alpha^2 v_\epsilon \left(2v_z/v_e - (v_z/v_e)^2 + \alpha^2 v_\epsilon v_{z|e}\right)}{1 + \alpha^2 v_{z|e} v_\epsilon} + \alpha^2 v_\theta, \qquad \frac{1}{1+\xi_I} = 1 + \alpha^2 v_\epsilon v_{z|e} \tag{A.23}$$

with $v_{z|e} \equiv \mathbb{V}ar\,[\tilde{z}|e_i] = (v_z^{-1} + v_u^{-1})^{-1}$ and $\alpha\mu = \frac{1}{2}\frac{\gamma(1)}{1-\gamma(1)}$. Moreover, traders are always better off in the no-information equilibrium with $\lambda = 0$ than in the full-information equilibrium with $\lambda = 1$, i.e.,

$$\frac{\mathcal{W}(0;e)}{\mathcal{W}(1;e)} = \exp\left\{-\frac{1}{2}\frac{\gamma(1)}{1-\gamma(1)} - \frac{1}{2}\alpha^2\left(\frac{v_u}{v_e}e\right)^2 v_\theta\right\}\sqrt{\frac{1+\alpha^2 v_\epsilon v_{z|e}}{1+\alpha^2(v_\epsilon + v_\theta)v_{z|e}}} \le 1. \tag{A.24}$$

A.10. **Proof of Proposition 4.4.** Given trader i's welfare in (4.8), the rate of change is given by

$$\frac{\mathcal{W}'(\lambda;e)}{-\mathcal{W}(\lambda;e)} = -\frac{1}{2}A'(\lambda)e^2 + \frac{1}{2}\left[\frac{\nu'(\lambda)}{\nu(\lambda)} - \frac{1-\lambda}{1-\lambda\gamma(\lambda)}\frac{v_U'(\lambda)}{v_U(\lambda)}\right] + \frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)} - \frac{\lambda\gamma'(\lambda)+\gamma(\lambda)}{2(1-\lambda\gamma(\lambda))^2}, \tag{A.25}$$

where $\nu(\lambda) = \mathbb{V}ar[\tilde{R}]$. Moreover, since $v_u \to \infty$, the equilibrium price P simplifies to (2.5), thus

$$A(\lambda) = \alpha^2 v_P(\lambda)/\frac{\nu(\lambda)}{v_U(\lambda)} > 0 \qquad \text{and} \qquad A'(0) = \frac{2n\xi_0}{(1+\xi_0)^2}. \tag{A.26}$$

The rest of the proof is the same as that for Corollary 3.1.

Appendix B. Existence and uniqueness of Nash equilibrium

This appendix examines the existence and uniqueness of the Nash equilibrium with respect to parameter μ, which measures the sensitivity to the cost of information acquisition. For convenience, we define $\xi_1 = \alpha^2 v_z v_\epsilon$. Note that $\xi_1 = \xi_I(1)$, representing the squared Sharpe ratio of informed traders when $\lambda = 1$. Intuitively, in equilibrium, $\lambda \to 0$ as $\mu \to \infty$; $\lambda = 1$ when μ is small enough; otherwise $\lambda \in (0,1)$. This is demonstrated as follows.

Proposition B.1. *Assume $c(p) = p^2$ and condition (2.11) holds. Then*

- (i) $\lambda = 0$ *as* $\mu \to \infty$*;*
- (ii) $\lambda = 1$ *when* $\mu \le \bar{\mu} := \frac{1}{2\alpha}\frac{\gamma_1}{1-\gamma_1}$*, where* $\gamma_1 \equiv \gamma(1) = 1 - \sqrt{\frac{n+\xi_1}{n+\xi_0}}$ *and* $\xi_1 = \alpha^2 v_z v_\epsilon$*;*
- (iii) *there exists a unique* $\lambda \in (0,1)$ *when* $\mu > \bar{\mu}$*.*

Moreover, the equilibrium price P satisfies (2.5) with the coefficients b_θ and b_z evaluated at the equilibrium λ.

Proof: Note that $\gamma(\lambda) \in (0,1)$. With $c(p) = p^2$, from the equilibrium condition $2\alpha\mu\lambda = \gamma(\lambda)/[1-\gamma(\lambda)]$, it is easy to see that $\lambda \to 0$ as $\mu \to \infty$. For $\lambda = 1$, we have $\mu = \bar{\mu} = \frac{1}{2\alpha}\frac{\gamma(1)}{1-\gamma(1)}$. It remains to discuss the case $\mu > \bar{\mu}$. To this aim, note that, in case of $c(p) = p^2$, the fixed point (2.13) is equivalent to

$$\lambda^2 - \frac{1}{\gamma(\lambda)}\lambda + \frac{1}{2\alpha\mu} = 0. \tag{B.1}$$

By defining

$$F_1(\lambda) = \frac{1}{2\gamma(\lambda)} - \frac{1}{2\gamma(\lambda)}\sqrt{1 - \frac{2\gamma^2(\lambda)}{\alpha\mu}}; \quad F_2(\lambda) = \frac{1}{2\gamma(\lambda)} + \frac{1}{2\gamma(\lambda)}\sqrt{1 - \frac{2\gamma^2(\lambda)}{\alpha\mu}},$$

(B.1) can be rewritten as $[\lambda - F_1(\lambda)][\lambda - F_2(\lambda)] = 0$. Assuming $\mu \geq 2\gamma^2(\lambda)/\alpha$ (otherwise the fixed point has no solution and $\lambda = 1$), F_1 and F_2 are well-defined. It is not difficult to show that $0 < F_1(\lambda) \leq F_2(\lambda)$. Therefore, since $F_1(0) > 0$, one solution to (B.1) exists if and only if $F_1(1) < 1$. This condition is exactly $\mu > \bar{\mu}$. Finally, concerning uniqueness, note that $dF_1(\lambda)/d\lambda < 0$. Indeed,

$$\frac{dF_1(\lambda)}{d\lambda} = -\frac{\gamma'(\lambda)}{2\gamma^2(\lambda)}\left(1 - \sqrt{1 - \frac{2\gamma^2(\lambda)}{\alpha\mu}}\right) + \frac{\gamma'(\lambda)}{\alpha\mu\sqrt{1 - \frac{2\gamma^2(\lambda)}{\alpha\mu}}} = \frac{\gamma'(\lambda)}{\gamma(\lambda)}\frac{F_1(\lambda)}{\sqrt{1 - \frac{2\gamma^2(\lambda)}{\alpha\mu}}} < 0.$$

Negativity is due to the fact that $\gamma'(\lambda) < 0$, $\gamma(\lambda) > 0$, and $F_1(\lambda) > 0$. By monotonicity, $\lambda = F_1(\lambda)$ provides at most one solution. Therefore, if a second solution $\tilde{\lambda}$ to the fixed point exists, it must solve $\tilde{\lambda} = F_2(\tilde{\lambda})$. By definition $F_2(\lambda) > \frac{1}{2\gamma(\lambda)}$; therefore, as soon as $\gamma(\lambda) < 1/2$, we have $\tilde{\lambda} = F_2(\tilde{\lambda}) > 1$, which is not feasible. This proves that the solution to the fixed point is unique as soon as the sufficient condition for (strict) concavity, $\gamma(\lambda) < 1/2$, is satisfied. □

Proposition B.1 provides a sufficient condition for the existence of a unique non-trivial Nash equilibrium $0 < \lambda < 1$. In general, the equilibrium fraction of informed traders is expected to increase as traders become less sensitive to the cost function. Put differently, we expect λ to be decreasing in μ. However, it turns out that such monotonicity is not guaranteed in general.

Proposition B.2. *The equilibrium $\lambda = \lambda(\mu)$ is decreasing in μ if and only if*

$$\frac{G'(\lambda)}{G(\lambda)} \leq \frac{c''(\lambda)}{c'(\lambda)}, \qquad G(\lambda) = \frac{\gamma(\lambda)}{1 - \lambda\gamma(\lambda)}; \tag{B.2}$$

or equivalently

$$\frac{\gamma^2(\lambda)+\gamma'(\lambda)}{1-\lambda\gamma(\lambda)} \le \frac{c''(\lambda)}{c'(\lambda)}. \tag{B.3}$$

For $c(p) = p^2$*, condition* (B.3) *becomes*

$$\lambda[\gamma^2(\lambda)+\gamma(\lambda)+\gamma'(\lambda)] \le 1. \tag{B.4}$$

In particular, at $\lambda = 0$*, condition* (B.4) *is always satisfied; while at* $\lambda = 1$*, it becomes*

$$\sqrt{\frac{\xi_1+n}{\xi_0+n}}\left[1+\frac{\xi_1+n}{\xi_0+n}\right] \le 3\frac{\xi_1+n}{\xi_0+n} + \frac{n^2\xi_1}{[\xi_0+n]^2}. \tag{B.5}$$

Proof: In equilibrium, $\alpha\mu g(\lambda) = -\frac{V_I(\lambda)-V_U(\lambda)}{V(\lambda)} = \frac{\gamma(\lambda)}{1-\lambda\gamma(\lambda)} = G(\lambda)$. For $\lambda = \lambda(\mu)$, taking the derivative w.r.t. μ, we have $\alpha c'(\lambda) = -\lambda'(\mu)[G'(\lambda) - \frac{c''(\lambda)}{c'(\lambda)}G(\lambda)]$. Therefore $\lambda'(\mu) \le 0$ if and only if (B.2) holds. Applying $c(p) = p^2$ to condition (B.2) leads to condition (B.3). Clearly, (B.3) holds for $\lambda = 0$. For $\lambda = 1$, condition (B.3) becomes $\gamma_1^2 + \gamma_1 + \gamma_1' \le 1$. Since $\gamma(\lambda) = 1 - f(\lambda)$, this is equivalent to $1 + f_1^2 \le 3f_1 + f_1'$. Using the fact that $f(\lambda) = \sqrt{\frac{\xi_1+n\lambda^2}{\xi_0+n\lambda^2}}$, we obtain condition (B.5). □

Proposition B.2 provides conditions for the equilibrium $\lambda = \lambda(\mu)$ to be decreasing in μ, or, put differently, it provides a less restrictive condition for the uniqueness of the Nash equilibrium λ. Note that, since $\lambda < 1$ and $\gamma'(\lambda) < 0$, condition (B.4) is always satisfied under condition (2.11). This leads to Proposition B.3.

Proposition B.3. *Consider the optimization problem* (2.7) *with* $c(p) = p^2$*. Suppose that* $\mu > \bar{\mu} := \frac{1}{2\alpha}\frac{\gamma_1}{1-\gamma_1}$ *and* $\gamma(\lambda) < 1/2$*, where* $\gamma_1 \equiv \gamma(1) = 1 - \sqrt{\frac{n+\xi_1}{n+\xi_0}}$*. Then, there exists a unique equilibrium* (P, λ) *such that (i)* $\lambda \in (0,1)$ *solves* (2.13) *and is decreasing in* μ*; and (ii)* P *is given by* (2.5).

The condition $\gamma(\lambda) < 1/2$ for the existence and uniqueness in Proposition B.3 indicates that the relative utility gain of being informed should be small. Note that $\gamma(\lambda) \le \gamma(0) = 1 - 1/\sqrt{1+n}$ due to $\gamma'(\lambda) < 0$.